‘The book of nature is written in the language of Mathematic.’

–**Galileo**

MATHEMATICS

LESSON PLANS

GRADE 10 TERM 1

MESSAGE FROM NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme provides most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Set aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook.

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online. Some useful web links are listed at the end of each lesson plan.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 1 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 1:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Algebraic expressions	3	Exponents and Surds	3	Sequences and Series	3
Exponents	2	Equations and Inequalities	3	Functions (including inverses and logarithms)	4
Number Patterns	1	Number patterns	2	Euclidean Geometry	2
Equations and Inequalities	2	Analytical Geometry	3	Trigonometry	2
Trigonometry	3				
Total	11	Total	11	Total	11

* Note: CAPS amendments to be implemented in January 2019 require that in Grade 12, Euclidean Geometry be done in Term 1. The Grade 12 lesson plans reflect this. In order to ensure that you have the full set of topics for Grade 12, we have included the Topic of Finance at the back of the Grade 12 lesson plans. Finance is NOT done in Term 1.

2. Term 1 lesson plans and assessments are provided for eleven weeks for all three grades.

3. Each week includes 4,5 hours of teaching time, as per CAPS.

4. You may need to adjust the lesson breakdown to fit in with your school's timetable.

LESSON PLAN STRUCTURE

The Lesson Plan for each term is divided into topics. Each topic is presented in exactly the same way:

TOPIC OVERVIEW

1. Each topic begins with a brief **Topic Overview**. The topic overview locates the topic within the term, and gives a clear idea of the time that should be spent on the topic. It also

indicates the percentage value of this topic in the final examination, and gives an overview of the important skills and content that will be covered.

2. The **Lesson Breakdown Table** is essentially the teaching plan for the topic. This table lists the title of each lesson in the topic, as well as a suggested time allocation.
For example:

	Lesson title	Suggested time (hours)
1	Revision	2,5
2	Venn diagrams	2,5
3	Inclusive and mutually exclusive events; Complementary and Exhaustive events	1,5
4	Revision and Consolidation	1,5

3. The **Sequential Table** shows the prior knowledge required for this topic, the current knowledge and skills to be covered, and how this topic will be built on in future years.
   - Use this table to think about the topic conceptually:
     - Looking back, what conceptual understanding should learners have already mastered?
     - Looking forward, what further conceptual understanding must you develop in learners, in order for them to move on successfully?
   - If learners are not equipped with the knowledge and skills required for you to continue teaching, try to ensure that they have some understanding of the key concepts before moving on.
   - In some topics, a revision lesson has been provided.

4. The **NCS Diagnostic Reports**. This section is potentially very useful. It lists common problems and misconceptions that are evident in learners' NSC examination scripts. The Lesson Plans aim to address these problem areas, but it is also a good idea for you to keep these in mind as you teach a topic.

5. The **Assessment of the Topic** section outlines the formal assessment requirements as prescribed by CAPS for Term 1 (page 54).

Grade	Assessment requirements for Term 1 (as prescribed in CAPS)
10	Project/ Investigation; Test
11	Project/ Investigation; Test
12	Project/ investigation; Assignment; Test

The assessments are included in the Lesson plans and Resource Pack for each grade.

Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide.

Some of the Grade 12 assessments come from: *Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide. DBE, Pretoria*

A team of experts comprising teachers and subject advisors from different provinces was appointed by the DBE to develop and compile the assessment tasks in this document. The team was required to extract excellent pieces of learner tasks from their respective schools and districts. The panel of experts spent a period of four days at the DBE developing tasks based on guidelines and policies. Moderation and quality assurance of the tasks were undertaken by national and provincial examiners and moderators to ensure that they are in line with CAPS requirements.
Mathematics School-based Assessment Exemplars – CAPS. Grade 12 Teacher Guide. DBE, Pretoria, p4

You can access this document from various sites, including:
https://www.education.gov.za/SchoolBasedAssessmentTasks2014/tabid/611/Default.aspx

6. The glossary of **Mathematical Vocabulary** provides an explanation of each word or phrase relevant to the topic. In some cases, an explanatory sketch is also provided. It is a good idea to display these words and their definitions or sketches somewhere in the classroom for the duration of the topic. It is also a good idea to encourage learners to copy down this table in their free time, or alternately, to photocopy the Mathematical Vocabulary for learners at the start of the topic. You should explicitly teach the words and their meanings as and when you encounter these words in the topic.

INDIVIDUAL LESSONS

1.. Following the **Topic Overview**, you will find the **Individual Lessons**. Each lesson is structured in exactly the same way. The routine within the individual lessons helps to improve time on task, and therefore, curriculum coverage.

2. In addition to the lesson title and time allocation, each lesson plan includes the following:

   A. Policy and Outcomes. This provides the CAPS reference, and an overview of the objectives that will be covered in the lesson.

   B. Classroom Management. This provides guidance and support as you plan and prepare for the lesson.

   - Make sure that you are ready to begin your lesson, have all your resources ready (including resources from the Resource Pack), have notes written up on the chalkboard, and are fully prepared to begin.

- Classroom management also suggests that you plan which textbook activities and exercises will be done at which point in the lesson, and that you work through all exercises prior to the lesson.
- In some cases, classroom management will also require you to photocopy an item for learners prior to the lesson, or to ensure that you have manipulatives such as boxes and tins available.

The Learner Practice Table. This lists the relevant practice exercises that are available in each of the approved textbooks.

- It is important to note that the textbooks deal with topics in different ways, and therefore provide a range of learner activities and exercises. Because of this, you will need to plan when you will get learners to do the textbook activities and exercises.
- If you feel that the textbook used by your learners does not provide sufficient practice activities and exercises, you may need to consult other textbooks or references, including online references.
- The *Siyavula* Open Source Mathematics textbooks are offered to anyone wishing to learn mathematics and can be accessed on the following website: https://www.everythingmaths.co.za/read

C. Conceptual Development*:*

This section provides support for the actual teaching stages of the lesson.

Introduction: This gives a brief overview of the lesson and how to approach it. Wherever possible, make links to prior knowledge and to everyday contexts.

Direct Instruction: Direct instruction forms the bulk of the lesson. This section describes the teaching steps that should be followed to ensure that learners develop conceptual understanding. It is important to note the following:

- Grey blocks talk directly to the teacher. These blocks include teaching tips or suggestions.
- Teaching is often done by working through an example on the chalkboard. These worked examples are always presented in a table. This table may include grey cells that are teaching notes. The teaching notes help the teacher to explain and demonstrate the working process to learners.
- As you work through the direct instruction section, and as you complete worked examples on the chalkboard, ensure that learners copy down:
    - formulae, reference notes and explanations
    - the worked examples, together with the learner's own annotations.
- These notes then become a reference for learners when completing examples on their own, or when preparing for examinations.
- At relevant points during the lesson, ensure that learners do some of the Learner Practice activities as outlined at the beginning of each lesson plan. Also, give

learners additional practice exercises and questions from past papers as homework. Ensure that learners are fully aware of your expectations in this respect.

D. ***Additional Activities / Reading***. This section provides you with web links related to the topic. Get into the habit of visiting these links as part of your lesson preparation. As teacher, it is always a good idea to be more informed than your learners. If possible, organise for learners to view video clips that you find particularly useful.

TRACKER

1. A Tracker is provided for each grade. The Trackers are CAPS compliant in terms of content and time.
2. You can use the Tracker to document your progress. This helps you to monitor your pacing and curriculum coverage. If you fall behind, make a plan to catch up.
3. Fill in the Tracker on a daily or weekly basis.
4. At the end of each week, try to reflect on your teaching progress. This can be done with the HoD, with a subject head, with a colleague, or on your own. Make meaningful notes about what went well and what didn't. Use the reflection section to reflect on your teaching, the learners' learning and to note anything you would do differently next time.
5. These notes can become an important part of your preparation in the following year.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 1.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating them, or by covering them in contact adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by:
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.
7. Add other resources to your resource file as you go along.
8. Note that these resources remain the property of the school to which they were issued..

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 1, Topic 1: Topic Overview

ALGEBRAIC EXPRESSIONS

A. TOPIC OVERVIEW

A

- This topic is the first of five topics in Term 1.
- This topic runs for three weeks (13,5 hours).
- It is presented over ten lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 13,5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Algebra (and Equations) counts 30% of the final Paper 1 examination.
- Algebra forms the foundation for all topics in Mathematics. It prepares learners for both Calculus and Statistics.
- It helps to develop critical thinking skills, including problem solving, logic, patterns and deductive reasoning.

Breakdown of topic into 10 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	The Real number System	2	6	Trinomials	2
2	Products	1	7	Sum/Difference of two cubes	1
3	Highest Common Factor	1.5	8	Algebraic Fractions: Simplification, multiplication and division	2
4	Investigation	1	9	Algebraic Fractions: Addition and Subtraction	1
5	Difference of two squares	1	10	Revision and Consolidation	1

B

SEQUENTIAL TABLE

Senior phase	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Identify variables, constants, exponents and coefficients • Identify and classify like and unlike terms and add and subtract them • Multiply monomials and binomials by monomials, binomials and trinomials • Divide monomials, binomials and trinomials by monomials • Determine squares, square roots, cubes and cube roots of algebraic terms • Factorise expressions to include taking out the HCF, difference of two squares and trinomials • Simplify algebraic fractions using factorisation.	• Understand that real numbers can be rational or irrational • Establish between which two integers a surd lies • Round real numbers to an appropriate degree of accuracy • Multiply a binomial by a trinomial • Factorise trinomials • Factorise the sums and difference of two cubes • Factorise by grouping in pairs • Simplify, add and subtract algebraic fractions with denominators up to cubes (sum and difference of).	• Non-real numbers • Nature of roots • Apply the laws of exponents to expressions involving rational exponents • Add, subtract, multiply and divide simple surds • Logarithms • Remainder theorem • Factor theorem • Factorise third degree polynomials.

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Algebraic Expressions.

These include:

- incorrect substitution
- the incorrect use of the words ‘and’ and ‘or’
- little understanding of interval notation and set builder notation
- incorrect rounding
- omitting the middle term when squaring a binomial.

It is important that you keep these issues in mind when teaching this section.

While teaching Algebra, it is important to revise and revisit the exponential laws – not only during this section but throughout the year. Factorisation needs to be taught intensively.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test, with memorandum, and an investigation, with a rubric are provided in the resource booklet. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
  Note: According to the short term curriculum changes as required by DBE, the investigation must be done in Term 1, and it must be done under supervision.
- The questions usually take the form of algebraic expressions and fractions that need to be simplified or factorised.
- Monitor each learner’s progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

C MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
real numbers	Measurable numbers that have a concrete value. Real numbers can be manipulated. For example, a common fraction can be changed into a decimal fraction.
rational numbers	A number that can be written in the form $\frac{a}{b}$ where $a;b \in Z$ but $b \neq 0$
irrational numbers	A number that cannot be written in the form $\frac{a}{b}$ For example, π or $\sqrt{10}$
integers	A number with no fractional part. Integers include the whole numbers as well as the negative numbers. $Z \in \{\ldots -3 ; -2 ; -1 ; 0 ; 1 ; 2 ; 3 \ldots.\}$
whole numbers	All positive numbers, including zero, which do not include any fractional parts $N_0 \in \{0 ;1 ; 2 ; 3 \ldots.\}$
natural numbers	Counting numbers $N \in \{1 ; 2 ; 3 \ldots.\}$
surd	Irrational numbers which are roots of positive integers and the exact value of roots can't be determined
expression	A mathematical formula which can include variables (letters), constants and operations Example: $2b + 3c$
simplify	Multiply using the distributive law and/or collect like terms in an expression
product	Answer to a multiplication question
sum	Answer to an addition question
difference	Answer to a subtraction question

quotient	Answer to a division question
term	Part of an algebraic expression Terms are separated by '+' or '–' signs
coefficient	A number or symbol (including its sign) multiplied with a variable in a term Example: $-2a$ is the coefficient of b in the term $-2ab$
variable	Letters of the alphabet which could represent different values Example: In the expression $m + 2$, m is a variable and could be replaced by a number to calculate the answer when m is equal to that specific number Variables can change values
constant	A number making up a term on its own in an expression Example: $a + 3b - 10$: -10 is the constant Constants cannot change value
substitution	Replacing a variable (letter of the alphabet) with a number to perform a calculation Example: If $b = 3$, the $b + 2 = 3 + 2 = 5$
monomial	One term expression
binomial	Two term expression
trinomial	Three term expression
polynomial	More than one term expression (two or more)
like term	Terms that have the same variables Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and $10abc$ are like terms and can be added or subtracted
unlike term	Terms that do not have the same variables Example: $3a$ and $2b$ are unlike terms and cannot be added or subtracted x and y are unlike terms and cannot be added or subtracted
exponent	In the example a^2, '2' (or squared) is the exponent. It is the number or variable written at the top of the base in smaller font

FOIL	Acronym short for: First, Outer, Inner, Last A method for multiplying a binomial by a binomial
factorisation	Factorisation or factoring consists of writing a number or an expression as a product of two or more factors. The inverse operation of simplifying and finding the product of expressions

TERM 1, TOPIC 1, LESSON 1

THE REAL NUMBER SYSTEM

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners should be able to:

- classify real numbers (including representing them on a number line, in set builder notation and interval notation)
- establish between which two integers a simple surd lies
- round real numbers.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resources 1 and 2 from the Resource Pack ready for use in this lesson.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson ensure the board with the heading is completely blank so the entire number system can be represented as a discussion with learners takes place.
6. The table on the next page provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 - 6	3 - 11	1 2 3 28	7 8 11 40	1.1 - 1.4	3 - 10	1.1	4	1.1 1.2 1.3	10 13 16

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

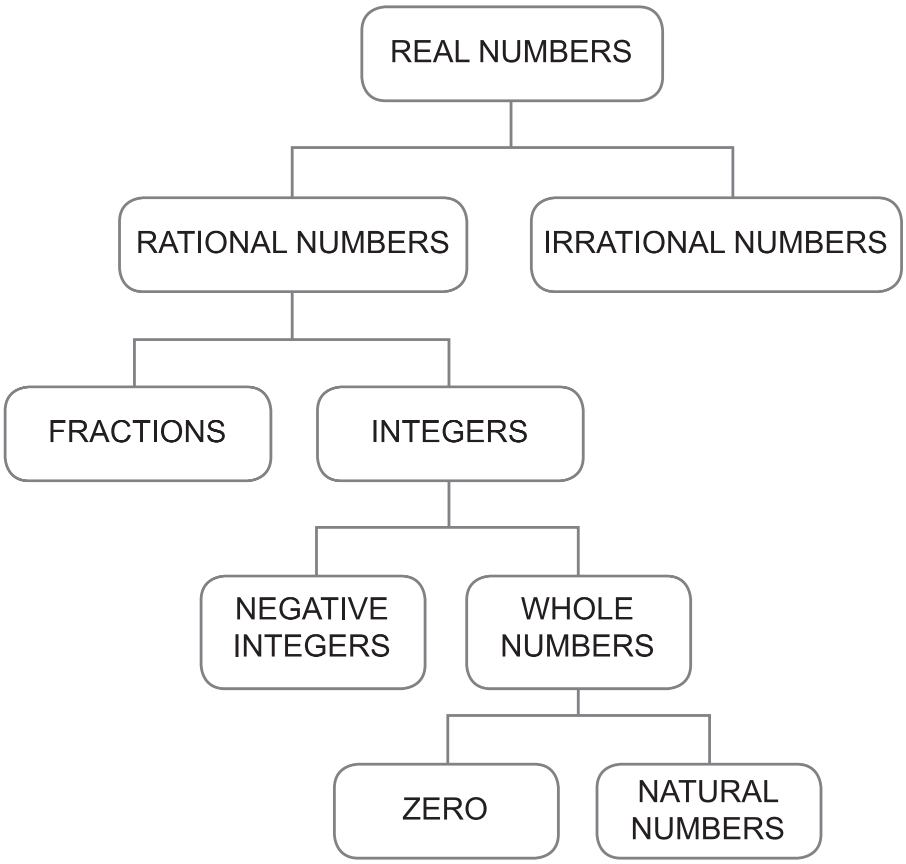
1. Most of this lesson should be revision for work already covered in previous years.
2. The textbooks approach this topic quite differently. It is important that you look carefully at your textbook and plan when you should stop teaching and give learners an exercise to do. The number of exercises available in the different textbooks ranges from one to six. As a result, there is no statement at the end of the lesson plan to give learners an exercise to do — ensure your learners have completed all the exercises in the textbook used before moving on to lesson 2.

DIRECT INSTRUCTION

1. Start the lesson by asking learners what they recall about the real number system. Give learners an opportunity to give you information. Ask questions such as:
   - Are all numbers real?
     (No – there are non-real numbers. For example, the square root of a negative number).
   - Can a number belong to more than one category?
     (Yes).
   - What were the first numbers you ever learned?
     (Natural numbers).

It may be beneficial to discuss with learners the time they were taught to count, (probably by a family member). Remind them that at that stage in their lives they only thought the numbers 1, 2, 3… existed. Soon after, they would have realised that zero was in fact a number. If they had 2 sweets and ate them, they would have started learning that 'none' was in fact zero in the number system. This extended their knowledge of numbers from natural numbers to whole numbers without realising it at this young age. The next step may have been to discover that when the temperature drops below freezing that a negative number is required or to take a lift and go into the basement parking also required a negative number – and their knowledge of the real number system now stretched to include integers too. Then they may have learned to share a biscuit and get half each or share a small cake and get a quarter. This extended their knowledge to include rational numbers. More recently, in the Senior Phase, they were introduced to pi while learning about circles as well as square roots and cube roots of non-perfect squares and cubes. This brings them to the present where they now have a knowledge of the entire Real Number System.

2. Take the time to draw the following two depictions of the real number system. Discuss them as you write on the board. Tell learners to copy them into their books as you are drawing them and explaining. (This is Resource 1 in the Resource Pack).

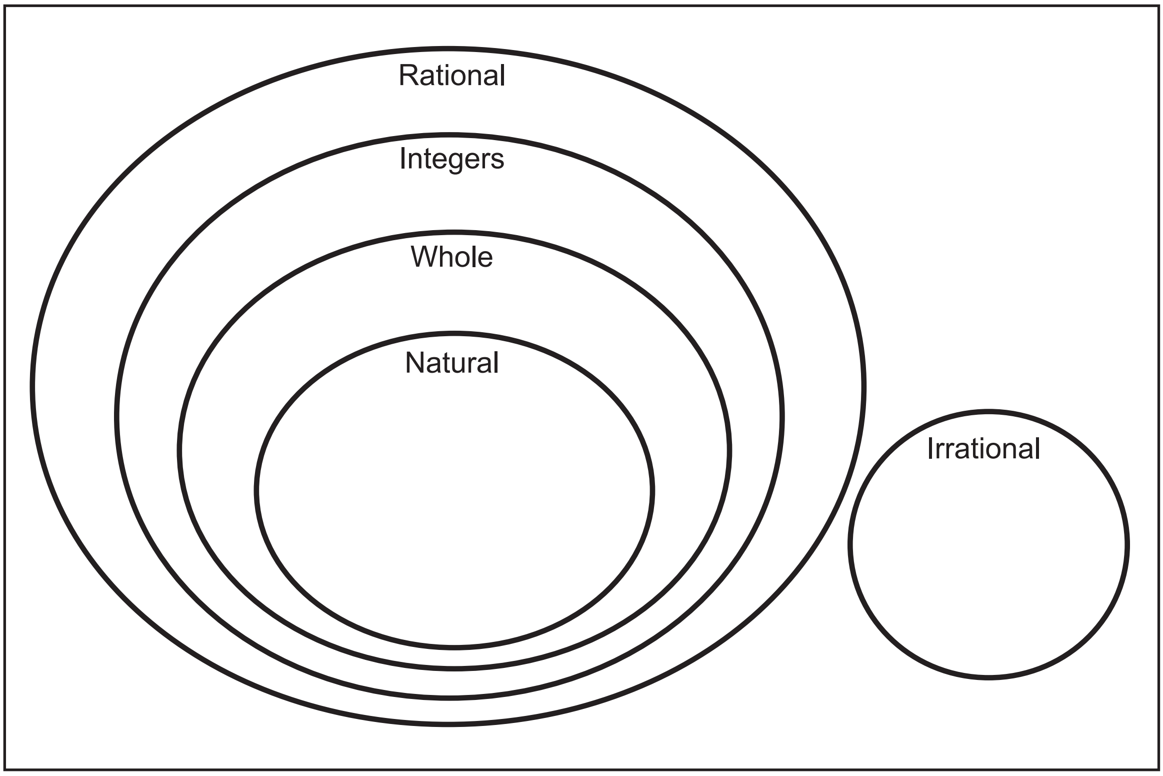


Explain how all the categories below a category are part of the category directly above. For example, integers are rational and real or irrational numbers are real.

3. Discuss the definition of a rational number. Say it a few times, then slowly so that learners can write in their books.
   Rational numbers are numbers that can be represented in the form $\frac{a}{b}$, where a and b are both integers but $b \neq 0$.

4. The following depiction of the real number system shows how a category may lie within another category. (This is Resource 2 in the Resource Pack).

REAL NUMBER SYSTEM



Give learners an opportunity to fill some numbers in the appropriate categories. Ask for volunteers to come to the board and fill the following numbers in the appropriate places:

$$-12\ ;\ \frac{1}{2}\ ;\ 0\ ;17\ ;\ \sqrt[3]{15}$$

Once this has been completed, discuss:

- 17 is natural as well as whole, an integer, rational and real
- 0 is whole, an integer, rational and real (it is outside the natural set as it is not a natural number)
- –12 is an integer, rational and real (it is outside the whole and natural set as it is not a whole number or a natural number)
- $\frac{1}{2}$ is rational and real (it is outside the integer, whole and natural numbers set as it belongs to neither of these)
- $\sqrt[3]{15}$ is irrational and real.

5. Important points to tell learners to assist them in their understanding:
   - The more common a number is, the more classifications it can belong to.
     For example, the number 5 is real, rational, an integer, whole and natural. Whereas π (which is a number that was only learned about more recently) is only real and irrational.
   - If a calculation is possible on a calculator (without giving an error message), the numbers used must be real. (Note: a scientific calculator would give a result with i to show that the number is imaginary and therefore non-real. Although we only need to deal with the Real number system at this stage there is, in fact, a complex number system which includes both real and non-real numbers).
   - Irrational numbers such as π, square roots of non-perfect squares and cube roots of non-perfect cubes ARE real (learners often confuse irrational with non-real).

REPRESENTING REAL NUMBERS ON NUMBER LINES

6. Say: *When representing inequalities on a number line, it is important to note what kinds of numbers are being represented.*

7. Say:
   - *if the numbers are natural, whole or integers, they will be represented by points on the number itself*
   - *if the numbers are real then a solid line is used to show that ALL the numbers including every possible number between integers are included.*

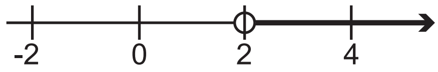
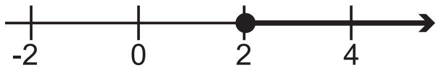
8. Write the following on the board and ask learners to write it in their books:

Inequality sign	words	Open/closed dot
>	Greater than	Open $\circ\!\!\longrightarrow$
≥	Greater than or equal to	Closed $\bullet\!\!\longrightarrow$
<	Less than	Open $\longleftarrow\!\!\circ$
≤	Less than or equal to	Closed $\longleftarrow\!\!\bullet$

As you are writing, explain that the open dot shows that a certain number is NOT included in the list whereas a closed dot is used to show that a certain number IS included in the list.

9. Point out that the direction of the inequality is the same as the direction of the arrow.

10. Use the following examples to explain further. Do them one at a time and discuss each one as it is completed. Remind learners what the open and closed dots mean.

Inequality	
$x > 2$	
$x \geq 2$	
$2 \leq x \leq 6$	
$2 < x < 6$	
$2 \leq x < 6$	
$2 < x \leq 6$	

11. Add a column on the right side of the table with the heading 'interval notation'.
Discuss $x > 2$. Ask: *If x represents an integer, what could x be?*
(3 ; 4 ; 5 ; 6….)
Ask: *Can you see that it is possible to list the possibilities?*
Say: *Now consider what would change if x represented real numbers.*
Ask: *What could x be now?*
Note that the above list is still possible, but learners must realise that there are so many more now. For example, 4,2 ; $\sqrt{20}$; π etc.
Say: *It is no longer possible to list the possibilities. For this reason we need to a way to show that all the real numbers are included in what we would like to list. Interval notation is used in place of a list.*

12. Complete the interval notation column that was added to the table. Explain as you complete each row.

Interval notation	Teaching notes:
$x \in (2 ; \infty)$	Note the round bracket to show that the number represented is NOT included. On the number line this is represented with an open dot. In the inequality this is represented with a < or >. Note the square bracket to show that the number represented IS included. On the number line this is represented with a closed dot. In the inequality this is represented with a ≤ or ≥.
$x \in [2 ; \infty)$	
$x \in [2 ; 6]$	
$x \in (2 ; 6)$	
$x \in [2 ; 6)$	
$x \in (2 ; 6]$	

SIMPLE SURDS

13. Discuss the definition of the word surd with learners: A surd is an irrational number which is the root of a positive integer, the exact value of roots can't be determined.
For example, $\sqrt{12}$, $\sqrt[3]{15}$. Tell learners to write this into their books.

14. Write $\sqrt{30}$ on the chalkboard. Ask learners if they can estimate the answer. After a few answers, ask learners to explain the strategy they used to estimate an answer.

15. Emphasise that the key point is to find the perfect squares that lie on either side of the number mentioned. In this case 25 and 36 lie on either side of 30.

$\sqrt{25} < \sqrt{30} < \sqrt{36}$

16. The final step should be to find the square roots of the perfect squares:

$5 < \sqrt{30} < 6$

17. Tell learners that they would not usually be asked to estimate an answer but rather to say between which two integers a surd lies. $\sqrt{30}$ lies between 5 and 6.

18. Do an example using a cube root with learners. Write $\sqrt[3]{100}$ on the chalkboard.
Ask: *What are the perfect cubes that lie on either side of 100?*
(64 and 125).

> If necessary, write the first 12 perfect squares and the first 5 perfect cubes on the chalkboard for learners. Learners are expected to know these well.

$$\sqrt[3]{64} < \sqrt[3]{100} < \sqrt[3]{125}$$

$$\therefore 4 < \sqrt[3]{100} < 5$$

The cube root of 100 lies between 4 and 5.

19. Ensure learners wrote both examples in their books.

ROUNDING REAL NUMBERS

20. Rounding is the process of making a number simpler but keeping its value close to what it was. The result is less accurate, but easier to use. Rounding numbers is an important skill, particularly when dealing with money and a quick estimate is required. Many questions give the number of decimal places that an answer should be rounded to. If rounding s is done incorrectly, learners could lose marks.

21. Remind learners of the rule to round – if the digit AFTER (to the right) the position we need to round to is less than 5, then we leave the digit in the position of interest as it is. If the digit AFTER the position we need to round to is 5 or more, then we round the digit in the position of interest UP.

22. Write the following decimal fraction on the chalkboard to use as an example:

23,871639

Round to TWO decimal places	Position to check – 3rd after the decimal comma It is a '1' so the digit in position 2 will not change	23,87
Round to THREE decimal places	Position to check – 4th after the decimal comma It is a '6' so the digit in position 2 will change – it will round UP	23,872
Round to the nearest whole	Position to check – 1st after the decimal comma It is an '8' so the digit in the units position will change – it will round UP	24

23. Give learners the following examples to do on their own:

Round 12,8341 to TWO decimal places	12,83
Round 1,6295 to TWO decimal places	1,63
Round 31,12348 to FOUR decimal places	31,1235
Round 112,0013 to TWO decimal places	112,00
Round 9,801 to the nearest whole number	10

24. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

25. Walk around the classroom as learners do the various exercises available in their textbook throughout the lesson. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=qkxLNSwop7c

http://learn.mindset.co.za/resources/mathematics/grade-10/algebraic-expressions/basics-algebraic-expressions/01-real-number-system

TERM 1, TOPIC 1, LESSON 2

PRODUCTS

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners should be able to:

- multiply a binomial with a binomial and a trinomial.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the examples from point 1 on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 2	14 15	4 5 6	13 13 15	2.1 2.2 w/sh	18 19 22	1.2 - 1.8	7-13	1.4	18

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have been multiplying algebraic expressions since Grade 8. Grade 10 work is an extension of all that they have done before.
2. Ensure learners are proficient in the algebra covered in the Senior Phase before moving on.

DIRECT INSTRUCTION

1. Start the lesson by asking learners to complete the following examples on their own to ascertain what learners are confident with and what they may need more practice in.

1	$(x+4)(x-5)$
2	$(y-3)(y-6)$
3	$(x+3)(x-3)$
4	$(a+4)^2$

5	$(x-5)^2$
6	$3a(a+b)-2(a^2-ab)$
7	$-x^3(-x+xy)$
8	$\frac{10x^2-5x+5}{x}$

2. Walk around the class as learners are working to assist where necessary but also to establish learners' weak points and strong points.
3. Once learners have had sufficient time to complete the 8 questions, work through them and get learners to mark their own work so that they can see if and where they made mistakes. Point out key concepts and refer to any issues you noticed while walking around.

	Solutions:	Teaching notes:
1	$(x+4)(x-5)$ $= x^2-5x+4x-20$ $= x^2-x-20$	Say: *Multiply a binomial by a binomial, each term of one binomial needs to be multiplied by each term of the other binomial.* *FOIL is often used as an acronym for:* *First (each first term)* *Outer (the outer 2 terms* *Inner (the inner two terms)* *Last (the last 2 terms).* To assist learners in remembering all four multiplications that need to take place, say: *Watch your signs! Remember that the sign to the left of a term belongs to that term.*
2	$(y-3)(y-6)$ $= y^2-6y-3y+18$ $= y^2-9y+18$	

<table>
<tr><td>3</td><td>$(x + 3)(x - 3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$</td><td>Say: *Although this question is the same as the two already done, it differs in that the answer only has two terms once the like terms have been collected. It results in the difference of 2 squares.*
This occurs because the signs are different but the terms in each binomial are the same.</td></tr>
<tr><td>4</td><td>$(a + 4)^2$
$= (a + 4)(a + 4)$
$= a^2 + 4a + 4a + 16$
$= a^2 + 8a + 16$</td><td rowspan="2">Spend some time on these two questions. Learners are quick to want to give $a^2 + 16$ and $x^2 + 25$ (or worse $x^2 - 25$) as solutions.
Ask: *What does it mean to square an expression?*
(To multiply by itself).
Say: *Write out the expression to be squared twice to show clearly what needs to be done. You will then see that squaring each term is not the correct procedure.*
Ask: *What is special about the two trinomials produced in the solutions?*
(They are perfect square trinomials – due to squaring required in the simplifying procedure).</td></tr>
<tr><td>5</td><td>$(x - 5)^2$
$= (x - 5)(x - 5)$
$= x^2 - 5x - 5x + 25$
$= x^2 - 10x + 25$</td></tr>
<tr><td>6</td><td>$3a(a + b) - 2(a^2 - ab)$
$= 3a^2 + 3ab - 2a^2 + 2ab$
$= a^2 + 5ab$</td><td rowspan="2">Say: *The distributive law is required to simplify. Once this has been completed, like terms need to be collected.*
Remember: the sign to the left of a term belongs to the term.</td></tr>
<tr><td>7</td><td>$-x^3(-x + xy)$
$= x^4 - x^4y$</td></tr>
<tr><td>8</td><td>$\frac{10x^2 - 5x + 5}{x}$
$= \frac{10x^2}{x} - \frac{5x}{x} + \frac{5}{x}$
$= 10x - 5 + \frac{5}{x}$</td><td>Note: If any learners factorised the numerator to simplify, this is acceptable. Point out, however, that knowing how to divide a polynomial by a monomial is still a skill that will be used later on (in Calculus).</td></tr>
</table>

4. Say: *When simplifying algebraic expressions, it is important to know the rules of exponents well. These will be covered in more detail in the next topic this term but the basics are still required for this topic.*

   Ask learners to tell you the six basic laws/ rules of exponents. Ensure that each of the following are mentioned as well as verbal explanations given.
   Learners should write these laws/ rules of exponents in their books.

When:		then:
multiplying powers that have the same base	$a^3 \times a^5 = a^8$	keep the base and add the exponents
dividing powers that have the same base	$\frac{a^{10}}{a^2} = a^8$	keep the base and subtract the exponents
raising a power to another power	$(a^5)^2 = a^{10}$	multiply the exponents
more than one base is raised to a power	$(ab)^3 = a^3b^3$	the exponent belongs to each base
a base is raised to a negative exponent	$a^{-2} = \frac{1}{a^2}$	reciprocate and change the sign of the exponent
a base has an exponent of zero	$a^0 = 1$	It will equal 1

5. Say: *Now that we have revised finding the products of algebraic expressions and the laws of exponents, we are going to move on to the one new concept covered in Grade 10 – multiplying a binomial by a trinomial.*

6. Ask: *What is a binomial?*
   (A two-term expression)
   Ask: *What is a trinomial?*
   (A three-term expression).

7. Write the following example on the board:

   $(x + 2)(x^2 - 4x + 3)$

8. Remind learners that each term in one bracket must be multiplied by each term in the other bracket.
   Ask: *How many multiplications will we need to perform?*
   (6)
   Use curves to show each pair that needs to be multiplied:

$$(x + 2)(x^2 - 4x + 3)$$

9. Do the solution on the board with learners. Ask them to contribute. Remind learners that the sign to the left of a term belongs to a term.
   Solution:
   $x^3 - 4x^2 + 3x + 2x^2 - 8x + 6$
   $= x^3 - 2x^2 - 5x + 6$

10. Do two more examples with learners:

$(3y^2 - 2)(-y^3 + 4y^2 - 8y)$ $= -3y^5 + 12y^4 - 24y^3 + 2y^3 - 8y^2 + 16y$ $= -3y^5 + 12y^4 - 22y^3 - 8y^2 + 16y$
$(2a^2b + ab^4)(3a^2b - 2b^2 + 1)$ $= 6a^4b^2 - 4a^2b^3 + 2a^2b + 3a^3b^5 - 2ab^6 + ab^4$

11. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

12. Give learners an exercise to complete on their own.

13. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=8VZnJVA-Trs
(multiply binomials instantly)

https://www.youtube.com/watch?v=cwldONEmzNk

https://www.youtube.com/watch?v=JhMBN4jIG_Y

TERM 1, TOPIC 1, LESSON 3

FACTORISATION – HIGHEST COMMON FACTOR

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners should be able to:

- factorise an expression by taking out the highest common factor
- use grouping to factorise an expression.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the three examples from point 3.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3 7	17 25	7 12	16 21	3.1	24	1.9 1.10 1.11 1.12	15 16 18 18	1.5 1.7	21 24

Note: The 1st exercise in Platinum includes the difference of two squares which is covered in the following lesson.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have already done some factorising in Grade 9.
2. Factorising remains an integral part of Mathematics throughout the FET phase. Ensure that learners are proficient in their Grade 9 factorising before moving on to any new concepts.

DIRECT INSTRUCTION

1. Start the lesson by asking questions which will enable you to establish learners' prior knowledge.
   Ask: *What does it mean to factorise?*
   (Write an expression as a product of two or more factors. It is the inverse of finding the product of two or more expressions using the distributive law).
2. Say: *Today we are going to only focus on the first type of factorisation. Can you tell me what that is? It is what we must ALWAYS look for first when asked to factorise.*
   (Taking out the highest common factor).
3. Before doing some examples, ask learners to simplify the following:

$2x(x+4)$	$5x^2y(2x-3y+1)$	$-7ab(2+4a^2)$
Solutions:		
$= 2x^2 + 8x$	$= 10x^3y - 15x^2y^2 + 5x^2y$	$= -14ab - 28a^3b$

4. Keep the questions and solutions on the board and remind learners that when we factorise, the three solutions would be given, and the questions become the answers. In other words, factorising is the inverse operation to finding products and simplifying.
5. Discuss the first solution, $2x^2 + 8x$. There are two terms. When the highest common factor has been found, there should still be two factors in the 'left over' bracket.
   Discuss the second solution, $10x^3y - 15x^2y^2 + 5x^2y$. There are three terms. When the highest common factor has been found, there should still be three factors in the 'left over' bracket.
   Discuss the third solution, $-14ab - 28a^3b$. Note that the negative in front of the first term would be part of the highest common factor. This would mean a change of sign for the factors left over.

6. Do the following examples with learners. Learners should write them in their books.

Examples	Teaching notes:
$3a^3 + 9ab - 12b^2$ $= 3(a^3 + 3ab - 4b^2)$	For each question, ask: *How many terms are there?* *Is there a common factor to all the terms?* While filling in the bracket after writing the common factor down, ask: *What must I multiply the common factor by to get the original term back?*
$-20x^2y^2 + 25x^3$ $= -5x^2(4y^2 - 5x)$	
$10f^4g^2 - 6f^2g^3 + 2fg$ $= 2fg(5f^3g - 3fg^2 + 1)$	

7. Ask learners if there are any questions. Before doing grouping, learners should complete an exercise on taking a common factor out of a polynomial.

8. Once learners have completed an exercise do any corrections required on the board. Ensure everyone is confident in factorising by taking out a common factor.

FACTORISING BY GROUPING

9. Say: *Sometimes the common factor can be a common bracket containing the same terms.*

10. Write the following example on the board:

$$2a(a + b) - 3b(a + b)$$

11. Ask: *How many terms are there in this expression?*
(Two terms).
Ask: *what is common to these two terms?*
$(a + b)$

12. Tell learners that the entire bracket is the common factor. Write this down now and point out to learners that this is the common factor (even though it is in a bracket) and once we have written it down we still need to open a second bracket to show what will be left in each term once the highest common factor has been taken out.
$(a + b)($

13. Once you get to this point, ask:
What must the common factor be multiplied by to get back to the first term?
$(2a)$
Ask: *What must the common factor be multiplied by to get back to the second term?*
$(3b)$

14. Complete the example
$2a(a + b) - 3b(a + b)$
$= (a + b)(2a - 3b)$

15. Ask learners to try these on their own:

$4xy(2 - 3z) + 5z(2 - 3z)$
$8ab^2(5m - 2n) - 4ab(5m - 2n)$

16. Once learners have completed the examples in their books, write the solutions in full on the board. Learners should mark and correct their own work.

Examples	Teaching notes:
$4xy(2 - 3z) + 5z(2 - 3z)$ $= (2 - 3z)(4xy + 5z)$	For each question, ask: *How many terms are there?* *Is there a common factor to these terms?* While filling in the bracket after writing the common factor down, ask: *What must I multiply the common factor with to get the original term back?*
$8ab^2(5m - 2n) - 4ab(5m - 2n)$ $= (5m - 2n)(8ab^2 - 4ab)$ $= 4ab(5m - 2n)(2b - 1)$	

Note: leave the two questions on the board.

17. Say: *These types of questions can be asked at another level. Sometimes we need to group the terms ourselves before we get to a step that looks similar to the questions we have just completed.*

18. Write the following example on the board:

$$2a^2 + 2ab - 3ab - 3b^2$$

Ask: *How many terms does this expression have?*
(4)
Is there a common factor for the 4 terms?
(No).

19. Say: *This means we need to check if we can group the terms in order to find a common factor among pairs of terms.* It is important that learners understand what the aim is — to create less terms (in this case two) that will have a common factor of a bracket of terms. Point to each of the three example questions to show learners what the aim is when grouping and finding a common factor.

20. Go back to the example written on the board.

$$2a^2 + 2ab - 3ab - 3b^2$$

Cover up the last two terms with your hand and ask:
Do the first two terms have a common factor?
(Yes - $2a$)

Ask: *What would be left if you took 2a out?*
($a + b$)
Tell learners to remember this.
Cover up the first two terms with your hand and ask:
Do the last two terms have a common factor?
(Yes - 3b).
Ask: *What would be left if you took 3b out?*
($a + b$)

21. Ask: *Do you have the same expression 'left over' each time?*
    (Yes)
    Say: *This is what we want – it will lead to a question like the three we have already done.*

22. Do the next step on the board with learners:

$$2a^2 + 2ab - 3ab - 3b^2$$

$$= 2a(a + b) - 3b(a + b)$$

23. Say: *Note that this is the same question we did first. Complete it yourself.*

24. Do one more example with learners.

$$3xy - 3y + x^2 - x$$

    Repeat the steps covering up two terms at a time.
    Cover up the last two terms with your hand and ask:
    Do the first two terms have a common factor?
    (Yes - 3y).
    Ask: *What would be left if you took 3y out?*
    ($x - 1$).
    Tell learners to remember this.
    Cover up the first two terms with your hand and ask:
    Do the last two terms have a common factor?
    (Yes - x).
    Ask: *What would be left if you took x out?*
    ($x - 1$).

25. Complete the example, explaining each step again as you do so.

$$3xy - 3y + x^2 - x$$

$$= 3y(x - 1) + x(x - 1)$$

$$= (x - 1)(3y + x)$$

26. Give learners two more examples to try on their own. Once they are complete, do them in full on the board. Learners should mark and correct their own work.

$3x^2 - 3xy + 2x - 2y$	$6a + 3b - 2ab^2 - b^3$
$= 3x(x - y) + 2(x - y)$	$= 3(2a + b) - b^2(2a + b)$
$= (x - y)(3x + 2)$	$= (2a + b)(3 - b^2)$

27. If you think learners need more examples, do some from the textbook you use.

28. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

29. Give learners an exercise to complete on their own.

30. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=6PWXdE2hRO4

https://www.youtube.com/watch?v=oV1DQAb4w7E

https://www.youtube.com/watch?annotation_id=annotation_1740409739&feature=iv&src_vid=-hiG-JwMNNsM&v=Isn-gwWh_hw
(grouping)

https://www.youtube.com/watch?v=HnWkfSbAg9A
(grouping)

TERM 1, TOPIC 1, LESSON 4

INVESTIGATION

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners will have:

- completed an investigation on the factorising of a difference of two squares using a knowledge of numbers and geometry.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the investigation. The investigation, rubric and marking guide can be found as Resource 3 in the Resource Pack.
3. Make copies of the investigation for each learner.

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. An investigation is an activity that should lead a learner to a deeper understanding of a mathematical concept.
2. This investigation deals with factorising a difference of two squares.

DIRECT INSTRUCTION

1. Hand out the investigation to each learner.
2. Tell learners that they must work on their own and that they will have 1 hour to complete the investigation.

3. Tell learners the mark they receive will count 15% towards their school-based assessment mark for the end of the year.

MARKING RUBRIC

MARKING RUBRIC

	3	2	1	0
PART A				
Evidence that instructions were followed methodically	Learners had written measurements of their square as they proceeded through the steps correctly.	Learners seemed to proceed through the steps easily.	Learners needed some assistance	No attempt made to follow the instructions
Correct use of mathematical symbols and language	All mathematics written with no errors	An error made in the use of exponents OR the use of subtraction for step 4	An error made in the use of exponents AND the use of subtraction in step 4.	No solutions
Demonstrates a clear understanding of relationships in Algebra	No errors in any of the steps, particularly step 4	Steps 1 and 3 correct but step 4 incorrect	At least two steps incorrect	No solutions
PART B				
Evidence that instructions were followed methodically	It was clear that the learners had followed each step carefully and used knowledge gained as they proceeded.	Learners seemed to proceed through the steps with ease	Learners needed some assistance	No attempt made to folow the instructions
Correct use of mathematical symbols and language	All mathematics written with no errors			No solutions
Demonstrates a cldear understanding of relationships in Algebra	No errors in any of the steps A clear answer in step 8	No errors in any of the steps but step 8 was left out or vague and incorrect	Errors made in 2 or more steps.	Errors made in 4 or more steps.

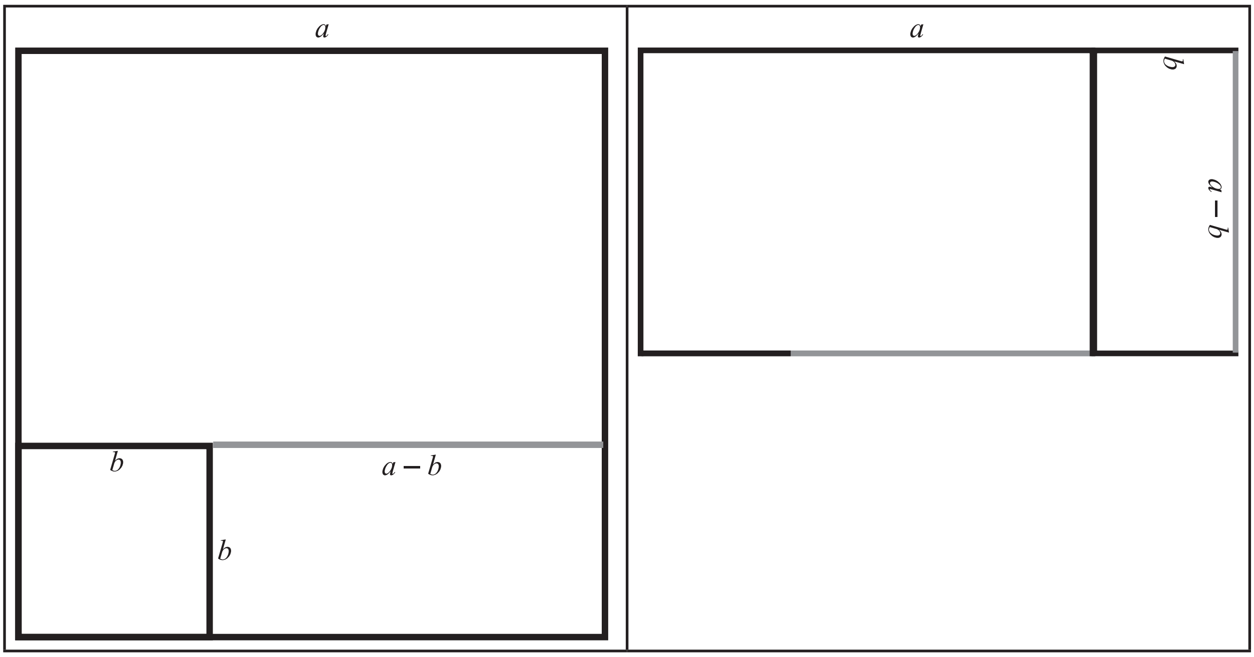
MARKING GUIDE:

PART A

1	a^2
3	b^2
4	$a^2 - b^2$

PART B

2	Length: $a - b$ Breadth: b
5	Length: $a + b$ Breadth: $a - b$
6	$(a + b)(a - b)$
7	They are equal
8	Factorising a difference of two squares $a^2 - b^2 = (a + b)(a - b)$



FACTORISATION – DIFFERENCE OF TWO SQUARES

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • factorise a difference of two squares.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the four questions from point 1.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	18	7	16	3.2 3.4	26 32	1.13 1.14	20 20	1.6	22

Note: The 2nd exercise in Classroom Maths incorporates grouping with the difference of two squares.

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have already covered this type of factorising in Grade 9.
2. More complicated examples will be covered in Grade 10. However, spend time ensuring learners are proficient in their Grade 9 factorising before moving on to any new concepts.

DIRECT INSTRUCTION

1. Start the lesson by asking learners to complete the following examples on their own to ascertain what learners are confident with and where they may need more practice.

1	$x^2 - 9$
2	$y^4 - 49$
3	$a^2b^2 - 100c^2$
4	$2f^2 - 50g^2$

2. Walk around as learners are working to assist where necessary but also to establish learners' weak points and strong points.
3. Once learners have had sufficient time to complete the eight questions, mark them with learners. Point out key concepts and refer to any issues you noticed while walking around. Learners should mark and correct their own work.

$x^2 - 9$ $= (x + 3)(x - 3)$	Start each question by asking, *How many terms are there? Is there a common factor in these terms?* (Remind learners that this should be the start of all factorising questions – if the answer is 'no', they should look for other types of factorising that have been learned). Remind learners of the name of this type of factorising: the difference (subtraction) of TWO squares. This only works if we have a square number subtract another square number. The two squares in question should be written as a product of their roots. The square root of each perfect square needs to be found. The signs need to be different for the roots to multiply and give the negative in front of the second term. This, in turn, ensures that the inner and outer term gained from multiplying out will equal zero.
$y^4 - 49$ $= (y^2 + 7)(y^2 - 7)$	
$a^2b^2 - \frac{1}{100}$ $=(ab + \frac{1}{10})(ab - \frac{1}{10})$	
$2f^2 - 50g^2$ $= 2(f^2 - 25g^2)$ $= 2(f + 5g)(f - 5g)$	

4. Say: *Remember that a SUM of two squares CANNOT be factorised.*

5. Remind learners that they should always check each factor to see if it can be factorised further.

6. For example: $2x^4 - 162$
   Ask: *How many terms? Is there a common factor?*
   (Two terms; yes – 2).

$$2x^4 - 162$$

$$= 2(x^4 - 81)$$

   Ask: *Can the terms in the bracket be factorised further?*
   (Yes – we have a difference of two squares).

$$= 2(x^2 + 9)(x^2 - 9)$$

   Ask: *Can any of the terms in the two brackets be factorised further?*
   (Yes – the second bracket has a difference of two squares in it).

$$= 2(x^2 + 9)(x + 3)(x - 3)$$

   Ask: *Can any of the terms in the three brackets be factorised further?*
   (No).

7. Ask learners to do some questions in the exercise provided in their textbook. Choose straightforward questions similar to those that have been done in the examples.

8. When learners have had time to complete the exercise, mark the work and make sure learners are confident in their ability to factorise a difference of two squares.

9. Write the following expression on the chalkboard:

$$(a + b)^2 - 144$$

   Ask: *How many terms are there? Is there a common factor?*
   (Two terms; no).
   Ask: *Are the two terms a difference of two squares?*
   (Yes).
   Say: *Although this question looks more complicated, we follow the same procedure as we did for the easier questions.*
   The roots of each term should be written as products.

   $\sqrt{(a+b)^2} = (a + b)$ and $\sqrt{144} = 12$

   $\therefore (a + b)^2 - 144 = ((a + b) + 12)((a + b) - 12)$

Point out that the bracket for the $a + b$ is not essential, but we are going to do another example where we could make an error if we didn't use the bracket.

10. Do the following example with learners:

$$4a^2 - (a - b)^2$$

Ask: *How many terms are there? Is there a common factor?*
(Two terms; no).
Ask: *Are the two terms a difference of two squares?*
(Yes).
Say: *The roots of each term need to be written as products:*

$\sqrt{4a^2} = 2a$ and $\sqrt{(a - b)^2} = (a - b)$

$\therefore 4a^2 - (a - b)^2 = (2a + (a - b))(2a - (a - b))$

This can be simplified further:

$$= (2a + a - b)(2a - a + b)$$

$$= (3a - b)(a + b)$$

11. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

12. Give learners an exercise to complete on their own.

13. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=_qyVzH3e1dY

https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-polynomials-3-special-product-forms/v/factoring-difference-of-squares

https://www.youtube.com/watch?v=BALzzQtchpk

TERM 1, TOPIC 1, LESSON 6

FACTORISATION – TRINOMIALS

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • factorise trinomials, including perfect square trinomials, and trinomials in the form $ax^2 + bx + c$ where $a \neq 1$.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the four questions from point 1.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5 6	20 23	10 11	20 20	3.3	29	1.15 – 1.9	21 – 26	1.8	27

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners encountered the factorising of trinomials in Grade 9. The type covered then will be revised briefly before going on to the new concepts covered in Grade 10.

DIRECT INSTRUCTION

1. Ask learners to factorise the following trinomials:

1	$x^2 + 7x + 10$
2	$x^2 - 7x + 12$
3	$x^2 - 2x - 15$
4	$x^2 + 7x - 18$

2. Walk around as learners are working to assist where necessary, and also to establish learners' weak points and strong points.

3. Once learners have had enough time to complete the four questions, mark them with learners. Point out key concepts and refer to any issues you noticed while walking around. Although steps are shown in the table, it is important to remind learners of the following in order to develop their conceptual knowledge:
   - The 'square' in the trinomial shows that there should be two factors – hence the two brackets at the start of the factorising process.
   - The signs are chosen to ensure that when multiplying the factors out (FOIL) the signs in the trinomial are correct.

4. Key issues to share with learners for factorising these trinomials:
   - When the last sign is addition, both signs are the same and match the middle term.
   - When the last sign is subtraction, both signs are different, and the larger factor chosen goes with the sign of the middle term.

$x^2 + 7x + 10$ $= (x + 5)(x + 2)$	• Open two brackets to represent the two factors. • Use the above rules to decide on the signs (in this case it will be + and + in each bracket). • Find the square root of the first term and put the solution at the front of each bracket. • Because the signs ARE THE SAME, ask: *What two numbers multiply to make the last term but also ADD UP to make the middle term?* (In this case 5 and 2). • Place these two factors at the back of each bracket. As the signs are the same it doesn't matter which number goes into each bracket.
$x^2 - 7x + 12$ $= (x - 4)(x - 3)$	• Open two brackets to represent the two factors. • Use the above rules to decide on the signs (in this case it will be - and - in each bracket). • Find the square root of the first term and put the solution at the front of each bracket. • Because the signs ARE THE SAME, (even though they are subtraction signs) ask: *What two numbers multiply to make the last term but also ADD UP to make the middle term?* (In this case 4 and 3). • Place these two factors at the back of each bracket. As the signs are the same it doesn't matter which number goes into each bracket.
$x^2 - 2x - 15$ $= (x - 5)(x + 3)$	• Open two brackets to represent the two factors. • Use the above rules to decide on the signs (in this case it will be - and + in each bracket. At this stage it does not matter which operator is placed in each bracket). • Find the square root of the first term and put the solution at the front of each bracket. • Because the signs ARE DIFFERENT, ask: *What two numbers multiply to make the last term but also SUBTRACT to make the middle term?* (In this case 5 and 3). • As the signs are different, it will matter where the numbers are placed. The biggest number ALWAYS goes with the sign in front of the middle term. In this case the 5 will go with the '–'.

$x^2 + 7x - 18$ $= (x + 9)(x - 2)$	• Open two brackets to represent the two factors. • Use the above rules to decide on the signs (in this case it will be - and + in each bracket. At this stage each one can be placed in either bracket. • Find the square root the first term and put the solution at the front of each bracket. • Because the signs ARE DIFFERENT, ask: *What two numbers multiply to make the last term but also SUBTRACT to make the middle term?* In this case 9 and 2. • As the signs are different, it will matter where the numbers are placed. The biggest number ALWAYS goes with the sign in front of the middle term (in this case the 9 will go with the '+').

5. Write the following questions on the board. Ask learners to multiply out and simplify:

1	$(x + 2)(x + 2)$
2	$(x - 5)(x - 5)$
3	$(a + 7)(a + 7)$
4	$(b - 10)(b - 10)$

6. Allow learners a few minutes to complete the four questions then mark and correct them. Solutions:

1	$x^2 + 4x + 4$
2	$x^2 - 10x + 25$
3	$a^2 + 14a + 49$
4	$b^2 - 10b + 100$

7. Ask: *What is the name of these special trinomials?*
(Perfect square trinomials).

8. Explain further, showing the reason for this name.
$(x + 2)(x + 2)$ could be written as $(x + 2)^2$ and $(x + 5)(x + 5) = (x + 5)^2$
Point out that the two factors are exactly the same. Therefore, when they are multiplied, they make a perfect square.

9. Give learners some perfect square trinomials to factorise.

1	$x^2 + 16x + 64$
2	$x^2 - 24x + 144$
3	$a^2 + 8a + 16$
4	$b^2 - 2b + 1$

Solutions:

1	$(x + 8)^2$
2	$(x - 12)^2$
3	$(a + 4)^2$
4	$(b - 1)^2$

10. If learners need more practice with this type of trinomial (leading coefficient of 1), give them an exercise to do now. If the Grade 10 book does not have enough questions, use an exercise from a Grade 9 book.

TRINOMIALS WITH A LEADING COEFFICIENT ≠ 1

11. Start this section by asking learners to multiply and simplify the following, reminding them that this should be quite easy for them now.

1	$(2x + 1)(x + 4)$	5	$(2x + 1)(x - 4)$
2	$(3x + 1)(2x + 3)$	6	$(3x + 1)(2x - 3)$
3	$(3y - 5)(y - 2)$	7	$(3y - 5)(y + 2)$
4	$(2y - 1)(5y - 2)$	8	$(2y - 1)(5y + 2)$

12. Walk around while learners are working. Assist where necessary and ascertain which learners are still be having difficulties with multiplying a binomial with a binomial.

13. Solutions:

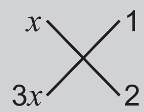
1	$2x^2 + 9x + 4$	5	$2x^2 - 7x - 4$
2	$6x^2 + 11x + 3$	6	$6x^2 - 7x - 3$
3	$3y^2 - 11y + 10$	7	$3y^2 + y - 10$
4	$10y^2 - 9y + 2$	8	$10y^2 - y - 2$

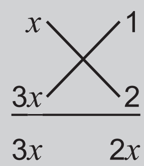
14. Ask learners to spend some time studying the factors and signs that made up the trinomials. Tell them that these are the trinomials we are going to learn to factorise now.

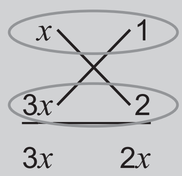
15. Say: *There is a little more to finding the correct factors and signs that will multiply out to form the trinomial being factorised. These will require plenty of practice.*

Write the following examples on the chalkboard, one at a time, and do them in full with the learners.

Learners should write the examples in their books and make notes as they do so.

16.
<table>
<tr><td colspan="2">Note:
Each of these types of trinomials should be factorised by starting the same way.
There will be two factors and therefore two brackets.
Look at the sign of the last term to decide on the signs that will be in the brackets.
The factors of the first and last term will be required to find the combination that will make the middle term, taking the signs into account.</td></tr>
<tr><td></td><td>Teaching notes and working to show learners:</td></tr>
<tr><td>$3x^2 + 5x + 2$</td><td>The last term is positive ∴ there needs to be two signs the same in the brackets
Second term positive ∴ (+)(+)
$3x^2 + 5x + 2 = (\quad + \quad)(\quad + \quad)$
Note that in this trinomial both the first and last terms are prime numbers. This is good - there can only be one combination of factors to multiply and result in the prime number.
Factors of term 1: $1x \times 3x$
Factors of last term: 1×2
Write the factors of each term underneath each other, placing a cross to show which terms will be multiplied to look for the correct combination:


Multiply and write the products underneath the term from the bottom row used.


Because the signs are the same these terms should add up to the middle term (5x). They do!
The horizontal factors can be put into the brackets as they are.


∴ $3x^2 + 5x + 2 = (x + 1)(3x + 2)$</td></tr>
</table>

$10x^2 + 19x + 6$	The last term is positive ∴ there needs to be two signs the same in the brackets Second term positive ∴ (+)(+) $10x^2 + 19x + 6 = (\quad + \quad)(\quad + \quad)$ Note that in this trinomial neither of the first and last terms are prime numbers. This means we will have to find the correct combination. Factors of term 1: $1x \times 10x$ and $2x \times 5x$ Factors of last term: 1×6 and 2×3 Choose what you think is the most likely pair. It is good practice to start with the pairs that are closest together (for the 1st term, rather 2 and 5 instead of 1 and 10) Note: Show learners what would happen if you had used the 2 at the top with the 2x and the 3 at the bottom. Remind learners that the two next to each other will be in one bracket. As there is no common factor in the trinomial, it makes no sense that the bracket could be made up of two terms that have a common factor which would need to be taken out. Therefore, that particular combination is impossible and not worth pursuing. $\begin{array}{cc} 2x & 3 \\ 5x & 2 \\ \hline 15x & 4x \end{array}$ Because the signs are the same these terms should add up to the middle term ($19x$). They do! The horizontal factors can be put into the brackets as they are. $\therefore 10x^2 + 19x + 6 = (2x + 3)(5x + 2)$ Tell learners they should always multiply out to check their answer is correct.

$5x^2 - 7x + 2$	The last term is positive ∴ there needs to be two signs the same in the brackets Second term negative ∴ (–)(–) $5x^2 - 7x + 2 = (\quad - \quad)(\quad - \quad)$ Note that in this trinomial both the first and last terms are prime numbers. This is good - there can only be one combination of factors to multiply and result in the prime number. Factors of term 1: $1x \times 5x$ Factors of last term: 1×2 $x \quad 1$ $5x \quad 2$ $5x \quad 2x$ Because the signs are the same these terms should add up to the middle term ($7x$). They do! The horizontal factors can be put into the brackets as they are. $\therefore 5x^2 - 7x + 2 = (x - 1)(5x - 2)$
$6x^2 - 13x + 6$	The last term is positive ∴ there needs to be two signs the same in the brackets Second term negative ∴ (–)(–) $6x^2 - 13x + 6 = (\quad - \quad)(\quad - \quad)$ Note that in this trinomial neither of the first and last terms are prime numbers. This means we will have to find the correct combination. Factors of term 1: $1x \times 6x$ and $2x \times 3x$ Factors of last term: 1×6 and 2×3 Choose what you think is the most likely pair. It is good practice to start with the pairs that are closest together. Remind learners that the 2's and 3's could not possibly be across from each other as this would create a common factor in the bracket which clearly does not exist in the trinomial. $2x \quad 3$ $3x \quad 2$ $9x \quad 4x$ Because the signs are the same these terms should add up to the middle term ($13x$). They do! The horizontal factors can be put into the brackets as they are. $6x^2 - 13x + 6 = (2x - 3)(3x - 2)$

Ask learners if they have any questions. Tell learners to look back at the first four questions you asked them to multiply out before doing the factorising and see for themselves whether they could use this method to factorise them.
Give learners an exercise to do at this stage before moving on to trinomials with a negative last term. Assist learners in becoming familiar with the process by spending time practicing factorising trinomials that all have a last term that is positive to.

$2x^2 - 5x - 12$

Last term negative ∴ need two different signs
Say: *We don't know yet what will go in the bracket with each sign so for now we will just use the fact that the signs need to be different. What goes in each bracket will be decided later.*

$$2x^2 - 5x - 12 = (\quad + \quad)(\quad - \quad)$$

Factors of term 1: $1x \times 2x$
Factors of last term: 1×12 and 3×4 and 2×6
Choose what you think is the most likely pair. Remember not to put two terms that have a common factor in the same horizontal line.

$$\begin{array}{ccc} x & & 4 \\ & \times & \\ 2x & & 3 \\ \hline 8x & & 3x \end{array}$$

Say: *At this stage the method changes a little because the signs are different in the brackets. Listen carefully.*
Because the signs are different, these terms should be subtracted to make the middle term ($5x$). They do!
This still does not help us to know which bracket to put each horizontal line of factors in.
Look at the middle term: $-5x$
Insert a positive and negative sign next to the answers of the cross multiplying to give this result ($-5x$)

$$\begin{array}{ccc} x & & 4 \\ & \times & \\ 2x & & 3 \\ \hline -8x & + & 3x \end{array}$$

The middle sign must be moved directly up; the sign on the left must be put in the upper row.

$$\begin{array}{ccc} x & - & 4 \\ & \times & \\ 2x & + & 3 \\ \hline -8x & + & 3x \end{array}$$

	The factors can now be placed into the correct brackets. $2x^2 - 5x - 12 = (2x + 3)(x - 4)$ Note: Some learners will be feeling a little lost — go through all the steps on the board before cleaning it to do the next example. Ask if anyone would like any further explanation. Assure learners that you are going to do two more examples and they will be able to ask more questions then.
$7x^2 + 3x - 4$	Last term negative ∴ need two different signs $7x^2 + 3x - 4 = (\quad + \quad)(\quad - \quad)$ Factors of term 1: $1x \times 7x$ Factors of last term: 1×4 and 2×2 Choose what you think is the most likely pair. Remember not to put two terms that have a common factor in the same horizontal line. (Note: In order to show learners that their first combination may not always work, we will start with one that doesn't work – don't point this out to learners until the check is done): $\begin{array}{cc} x & 2 \\ 7x & 2 \\ \hline 14x & 2x \end{array}$ Because the signs are different these terms should subtract to make the middle term ($3x$). They do not. We need to re-think the combinations. Normally, it would be best to first swop the second combination around, but as these are the same number that won't make any difference. We therefore need to try the other set of factors for the last term. $\begin{array}{cc} x & 1 \\ 7x & 4 \\ \hline +7x & -\ 4x \end{array}$ Because the signs are the different these terms should subtract to make the middle term ($3x$). They do! Now we need to confirm which pairs go in which brackets. Look at the middle term: $+3x$ Insert a positive and negative sign next to the answers of the cross multiplying to give this result ($+3x$) $\begin{array}{cc} x & 1 \\ 7x & 4 \\ \hline +7x & -\ 4x \end{array}$

	The middle sign must be moved directly up; the sign on the left must be put in the upper row. $\begin{array}{rcr} x & + & 1 \\ \underline{7x} & \underline{-} & \underline{4} \\ +7x & - & 4x \end{array}$ The factors can now be placed into the correct brackets: $7x^2 + 3x - 4 = (x + 1)(7x - 4)$
$24x^2 + 31x - 15$	Last term negative ∴ need two different signs $24x^2 + 31x - 15 = (\quad + \quad)(\quad - \quad)$ Factors of term 1: $1x \times 24x$ and $2x$ and $12x$ and $3x$ and $8x$ and $4x$ and $6x$ Factors of last term: 1×15 and 3×5 Choose what you think is the most likely pair. Remember not to put two terms that have a common factor in the same horizontal line. (Note: Allow learners to tell you what pairs they might choose. It would work out well if you could do one or two that didn't work out immediately to give learners practice in what to do when their chosen combination doesn't work). $\begin{array}{rr} 3x & 5 \\ \underline{8x} & \underline{3} \\ 40x & 9x \end{array}$ Because <u>the signs are different</u> these terms should <u>subtract</u> to make the middle term $(31x)$. They do! Now we need to confirm which pairs go in which brackets. Look at the middle term: $+31x$ Insert a positive and negative sign next to the answers of the cross multiplying to give this result $(+31x)$ $\begin{array}{rcr} 3x & & 5 \\ \underline{8x} & \underline{\quad} & \underline{3} \\ +40x & - & 9x \end{array}$ The middle sign must be moved directly up; the sign on the left must be put in the upper row. $\begin{array}{rcr} 3x & + & 5 \\ \underline{8x} & \underline{-} & \underline{3} \\ +40x & - & 9x \end{array}$ The factors can now be placed into the correct brackets: $24x^2 + 31x - 15 = (3x + 5)(8x - 3)$

17. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
18. Give learners an exercise to complete on their own.
19. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=fmLM8xt-YIg

https://www.youtube.com/watch?v=Fwjkor2y9kw

https://www.youtube.com/watch?v=AMEau9OE6Bs

TERM 1, TOPIC 1, LESSON 7

FACTORISATION – SUM AND DIFFERENCE OF TWO CUBES

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • factorise the sum and difference of two cubes.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the four questions from point.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
8	27	15 16	23 24			1.20	28	1.9	30

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This is a new concept for learners.
2. Provide opportunities for learners to first multiply expressions that lead to the sum or difference of two cubes and investigating the ideas and rules for themselves.

DIRECT INSTRUCTION

1. Ask learners to multiply and simplify the following expressions:

1	$(x-2)(x^2+2x+4)$
2	$(a+3)(a^2-3a+9)$
3	$(y-5)(y^2+5y+25)$
4	$(b+1)(b^2-b+1)$

2. Walk around the classroom assisting learners where necessary. Tell learners to use this opportunity to revise and practice finding products of expressions.
3. Once learners have finished the examples, give the answers and ask if anyone needs any of them doing in full on the chalkboard.

   Solutions:

1	x^3-8
2	a^3+27
3	y^3-125
4	b^3+1

4. Ask: *What kind of numbers are represented by each term in the answers?*
   (They are all perfect cubes).
   Say: *Notice that two of the expressions in the answers form a sum of two cubes and two of them form a difference of two cubes.*
5. Say: *Remember that factorising is the inverse of multiplying out and simplifying.*
   Say: *Note that that this exercise shows us that we can factorise a sum of two cubes and a difference of two cubes.*

6. Write each sum/difference of two cubes equal to its factorised answer.

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$	$a^3 + 27 = (a + 3)(a^2 - 3a + 9)$
$y^3 - 125 = (y - 5)(y^2 + 5y + 25)$	$b^3 + 1 = (b + 1)(b^2 - b + 1)$

7. Ask learners to look carefully at each expression and its factorised answer. Tell them to try to find the steps to follow to factorise a sum or difference of two cubes. After a few minutes, ask learners to discuss the method with a partner.
8. Ask who feels confident that they could explain the steps for factorising a sum or difference of two cubes. If anyone would like to try – allow him/her to do so.
9. Once the explanation has been given, praise the learners for their participation and for any part they did correctly. Ask learners to copy the following into their books:

Examples with steps:

$a^3 - 64$ $= (\qquad)(\qquad\qquad)$	Note that the two terms form a sum of two cubes. To factorise, open a 'binomial' bracket and a 'trinomial' bracket.
$a^3 - 64$ $= (a - 4)(\qquad\qquad)$	Cube root each term in the expression and keep the same sign to populate the first bracket.
$a^3 - 64$ $= (a - 4)(a^2 \qquad + 16)$	Square the first term in the first bracket to get the first term for the second bracket. Square the first term in the first bracket to get the first term for the second bracket. (Remember that when a number is squared the answer will always be positive).
$a^3 - 64$ $= (a - 4)(a^2 + 4a + 16)$	To complete the process and find the second term in the second bracket: Multiply the two terms in the first bracket but change the sign.

10. 10. Using a different example, explain the process again. Learners should copy both examples into their books.

$t^3 + 1000$

cube root and keep the sign

multiply terms in first bracket and change sign

$= (t + 10)(t^2 - 10t + 100)$

square 1st term to get new 1st term

square last term to get new last term

11. If you think it will assist your learners, you can share the following mnemonic with them to help them remember the three signs required.

SOAP	S - Same sign	O – opposite sign	AP – Always positive

12. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

13. Give learners an exercise to complete on their own.

14. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=RNGSFF4YZJ8

https://www.youtube.com/watch?v=A7omOLk9Sqg

https://www.youtube.com/watch?v=ktZOeCx_P_c

TERM 1, TOPIC 1, LESSON 8

ALGEBRAIC FRACTIONS – SIMPLIFICATION, MULTIPLICATION AND DIVISION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners will have revised:

- simplify one term algebraic fractions using factorisation
- multiply and divide algebraic fractions.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the first three examples.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
9 10 11	28 30 31	17	26	4.1 4.2	38 39	1.21 1.22	30 32	1.10 (1&2)	34

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This topic requires good factorising skills. If learners still need practice in this, then consider a remedial lesson (time permitting during the mathematics lesson) or an extra lesson.
2. Learners need to factorise and work with algebraic fractions throughout the FET – it is important that they are confident in the level required in Grade 10.

DIRECT INSTRUCTION

1. Remind learners how non-algebraic fractions are multiplied and divided. Do the following examples with them, reminding them of the steps to follow. This will allow learners to focus on the rules of multiplying and dividing fractions in general instead of needing to factorise algebraic expressions as well.
2. Examples and notes:

$\frac{3}{4} \times \frac{8}{9}$ $= \frac{1}{1} \times \frac{2}{3}$ $= \frac{2}{3}$	Say: *When multiplying fractions, the numerators should be multiplied with each other and the denominators should be multiplied with each other.* *However, if any numerator can be simplified with any denominator then this should be done as it will make the multiplication simpler.* Note: The step after the simplification process is not essential (here or in any of the examples that follow). Learners can simplify within the first step then go straight to the answer. It is merely shown here to show what should be left after simplifying has occurred.
$\frac{-5}{12} \times \frac{-3}{20}$ $= \frac{-1}{4} \times \frac{-1}{4}$ $= \frac{1}{16}$	
$\frac{4}{15} \div \frac{32}{35}$ $= \frac{4}{15} \times \frac{35}{32}$ $= \frac{1}{3} \times \frac{7}{8}$ $= \frac{7}{24}$	Say: *When dividing fractions, the division can be changed to multiplication and the fraction that follows the change will be reciprocated.* Note: many learners do not remember or understand why this is done – the first link at the end of this lesson is for a video explaining the reason we 'change to times and reciprocate'.

$\frac{9}{14} \div \frac{27}{2} \times \frac{28}{3}$ $= \frac{9}{14} \times \frac{2}{27} \times \frac{28}{3}$ $= \frac{1}{1} \times \frac{1}{3} \times \frac{4}{3}$ $= \frac{4}{9}$	Point out that in this question only one change and reciprocal needs to occur as there is only one division.

3. Tell learners that we need to apply these skills to algebraic fractions.

4. Write the following three examples on the board to do with learners:

$\frac{2x}{5y^2} \times \frac{25y^3}{32x^2}$ $= \frac{1}{1} \times \frac{5y}{16x}$ $= \frac{5y}{16x}$	Each of these questions is treated the same as the previous three examples. Simplify numerators with denominators where possible, and then multiply numerators with numerators and denominators with denominators. Division will be changed to multiplication and the fraction directly after the change of sign will be reciprocated.
$\frac{3}{5ab} \div \frac{9a}{25b^2}$ $= \frac{3}{5ab} \times \frac{25b^2}{9a}$ $= \frac{1}{a} \times \frac{5b}{3a}$ $= \frac{5b}{3a^2}$	
$\frac{10g^2h^3}{21} \div \frac{5g^4}{3} \times \frac{14}{gh}$ $= \frac{10g^2h^3}{21} \times \frac{3}{5g^4} \times \frac{14}{gh}$ $= \frac{2h^2}{1} \times \frac{1}{g^2} \times \frac{2}{g}$ $= \frac{4h^2}{g^3}$	

5. Say: *These examples should have been straightforward and that we are now going to do a few examples more relevant to Grade 10. With these examples, there will be expressions involving more than one term and the only way we will be able to simplify, will be to factorise the expressions found in the numerator and denominator positions.*

6. Write the following examples on the chalkboard.
   Learners should write them in their books, making notes as they do so.

$$\frac{p^2+p-2}{2p-4} \times \frac{3p-6}{2+p}$$

To multiply fractions, the basic rule is to multiply numerators with numerators and denominators with denominators.
However, as mentioned in the previous examples, we need to simplify numerators with denominators first where possible.
Point out that as the question stands now, no simplifying can occur as we are dealing with more than one term. We need to factorise first so that we can deal with one term made up of a product of factors which we can then simplify with other terms.
Go through each of the four possible expressions by pointing at each one and asking if any can be factorised and if so, ask for a brief explanation.

$p^2 + p - 2$: yes – it is a trinomial and will have two factors

$2p - 4$: yes – there is a common factor

$3p - 6$: yes – there is a common factor

$2 + p$: no – it is in its simplest form already

Ask learners to factorise for you as you complete the example on the board.

Solution:

$$\frac{p^2+p-2}{2p-4} \times \frac{3p-6}{2+p}$$

$$= \frac{(p+2)(p-1)}{2(p-2)} \times \frac{3(p-2)}{2+p}$$

$$= \frac{(p-1)}{2} \times \frac{3}{1}$$

$$= \frac{3(p-1)}{2}$$

Ask: *What can be simplified?*
(The $(p-2)$ and the $(p-2)$).
Ask: *Is* $(p+2) = (2+p)$*?*
Yes (if learners are unsure discuss whether $3 + 2$ gives the same answer as $2 + 3$).

$$\frac{x^2-4x+4}{3x+3} \div \frac{2x-4}{1-x^2}$$

The only significant difference between this question and the first question is that the division needs to be changed to multiplication and the fraction after the change needs to be reciprocated.

Once this step has been completed, go through each of the four possible expressions with learners. Point at each expression and ask:

Can this expression be factorised and if so, briefly explain.

$x^2 - 4x + 4$: yes – it is a trinomial and will have two factors

$3x + 3$: yes – there is a common factor

$1 - x^2$: yes – it is a difference of two squares

$2x - 4$: yes – there is a common factor

Ask learners to factorise for you as you complete the example on the board.

$$\frac{x^2-4x+4}{3x+3} \div \frac{2x-4}{1-x^2}$$
$$= \frac{x^2-4x+4}{3x+3} \times \frac{1-x^2}{2x-4}$$
$$= \frac{(x-2)(x-2)}{3(x+1)} \times \frac{(1+x)(1-x)}{2(x-2)}$$
$$= \frac{(x-2)}{3} \times \frac{(1-x)}{2}$$
$$= \frac{(x-2)(1-x)}{6}$$

$$\frac{5ab-5a}{2b^2+b-3} \times \frac{6-4b}{5b^2+10b+5} \div \frac{2a+4ab}{2b^2+3b+1}$$

After changing the division to multiplication and reciprocating the fraction that follows, proceed as with the previous examples.

$5ab - 5a$

Yes – there is a common factor

$2b^2 + b - 3$

Yes – it is a trinomial and will have two factors

$6 - 4b$

Yes – there is a common factor

$5b^2 + 10b + 5$

Yes – it is a trinomial that has a common factor (remind learners to ALWAYS look for a common factor first). Point out that once the common factor has been taken out, we will need to check if it factorises further. $2b^2 + 3b + 1$ Yes – it is a trinomial and will have two factors $2a + 4ab$ Yes – there is a common factor.
$\frac{5ab - 5a}{2b^2 + b - 3} \times \frac{6 - 4b}{5b^2 + 10b + 5} \div \frac{2a + 4ab}{2b^2 + 3b + 1}$ $= \frac{5ab - 5a}{2b^2 + b - 3} \times \frac{6 - 4b}{5b^2 + 10b + 5} \times \frac{2b^2 + 3b + 1}{2a + 4ab}$ $= \frac{5a(b - 1)}{(2b + 3)(b - 1)} \times \frac{2(3 - 2b)}{5(b^2 + 2b + 1)} \times \frac{(2b + 1)(b + 1)}{2a(1 + 2b)}$ $= \frac{5a(b - 1)}{(2b + 3)(b - 1)} \times \frac{2(3 - 2b)}{5(b + 1)(b + 1)} \times \frac{(2b + 1)(b + 1)}{2a(1 + 2b)}$ $= \frac{1}{(2b + 3)} \times \frac{(3 - 2b)}{(b + 1)} \times \frac{1}{1}$ $= \frac{(3 - 2b)}{(2b + 3)(b + 1)}$

7. Ask if anyone has any questions.
8. Before doing one final example:
   Say: *Although we have dealt with $p + 2 = 2 + p$ and $2b + 1 = 1 + 2b$, we need to look at what happens if a subtraction sign is involved.*
   Ask: *Is* 5 – 3 = 3 – 5?
   (No).
   Say: *It should be clear that* $x - 2 \neq 2 - x$
9. Show learners how to manipulate expressions such as these in order to simplify further.
   Write $2 - x$ on the chalkboard. Take out a '–1'.

   $$-1(-2 + x)$$

   Show that if the distributive law is used, the expression will equal what we started with $(2 - x)$
   Ask learners if they agree that $-2 + x$ is the same as $x - 2$.
   Show then, that $2 - x = -(x - 2)$
10. Say: *By taking out a negative 1 (and we only need to write the negative sign) and swopping the terms around we can manipulate the expression to allow us to factorise further.*

11. Write the following two examples on the board and ask learners to simplify the fractions using this concept:

1. $\frac{x-3}{3-x}$	2. $\frac{2b-3}{3-2b}$
Solutions:	
1. $\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -1$	2. $\frac{2b-3}{3-2b} = \frac{2b-3}{-(2b-3)} = -1$

12. Do one final example where this skill will be required. Learners should write it in their books. Discuss each step as you did before with the previous three examples.

Solutions:

$$\frac{ab-a^2}{a^2+2ab+b^2} \div \frac{a^2}{a^2-b^2} \times \frac{a^2+ab}{a^2-2ab+b^2}$$

$$= \frac{ab-a^2}{a^2+2ab+b^2} \times \frac{a^2-b^2}{a^2} \times \frac{a^2+ab}{a^2-2ab+b^2}$$

$$= \frac{a(b-a)}{(a+b)(a+b)} \times \frac{(a+b)(a-b)}{a^2} \times \frac{a(a+b)}{(a-b)(a-b)}$$

$$= \frac{(b-a)}{1} \times \frac{1}{1} \times \frac{1}{(a-b)}$$

$$= \frac{(a-b)}{1} \times \frac{1}{1} \times \frac{1}{(a-b)}$$

$= -1$

13. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

14. Summarise the basic steps to follow with learners and tell them to write them down in their books.

Multiplication and Division of Algebraic fractions:

- If division – change to multiplication and reciprocate the fraction after the change
- Factorise all numerators and denominators
- Simplify factors in any numerator with factors in any denominator where possible
- Multiply numerators with numerators and denominators with denominators

15. Give learners an exercise to complete on their own.

16. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=ybwz9OGGzng

(why we change divide to multiply and reciprocate in division of fractions)

https://www.youtube.com/watch?v=UXuYYjy_ZJU

TERM 1, TOPIC 1, LESSON 9

ALGEBRAIC FRACTIONS – ADDITION AND SUBTRACTION

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners should be able to:

- add and subtract algebraic fractions.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the first two examples.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
12 13	33 35	18 19	28 29	4.3	40	1.23 1.24 1.25	33 34 35	1.10 (3)	34

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Addition and subtraction of fractions is a skill required throughout the FET phase. It will be used in other topics such as Trigonometry and Calculus in Grade 11 and Grade 12.

DIRECT INSTRUCTION

1. Revise how to add and subtract fractions with simple non-algebraic fractions. This will allow learners to focus on the rules of adding and subtracting fractions in general instead of needing to deal with algebra.
2. Examples and notes:

$\frac{1}{4}+\frac{1}{2}$ $=\frac{2}{8}+\frac{4}{8}$ $=\frac{6}{8}$ $=\frac{3}{4}$	Share the following with learners: Adding and subtracting fractions can only be done if the denominators are the same. It is like only being able to add and subtract like terms. By making the denominators the same, we can add or subtract fractions of the same value. For example, adding quarters to quarters is relatively simple. Although it isn't essential, finding the LOWEST common denominator is easier to avoid larger numbers and more simplifying at a later stage.
$\frac{2}{9}-\frac{3}{5}$ $=\frac{10}{45}-\frac{27}{45}$ $=-\frac{17}{45}$	

3. Say: We need to apply these skills to algebraic fractions.

4. Write the following three examples on the board to do with learners:

1. $\frac{2x}{3}-\frac{3x}{12}$ $=\frac{2x(4)}{12}-\frac{3x}{12}$ $=\frac{8x-3x}{12}$ $=\frac{5x}{12}$	2. $\frac{3}{y}+\frac{1}{y^2}$ $=\frac{3(y)}{y^2}+\frac{1}{y^2}$ $=\frac{3y+1}{y2}$	3. $\frac{2}{5x}+\frac{1}{3x^3}-\frac{3}{15x^2}$ $=\frac{2(3x^2)}{15x^3}+\frac{1(5)}{15x^3}-\frac{3(x)}{15x^3}$ $=\frac{6x^2}{15x^3}+\frac{5}{15x^3}-\frac{3x}{15x^3}$ $=\frac{6x^2+5-3x}{15x^3}$

When finding the lowest common denominator, explain that although the term LCD could imply that the number will be small this is not the case. We just need to find the smallest/lowest number that ALL the denominators can go INTO.
Remind learners that only like terms in the numerator can be added or subtracted.
Remind learners what they are doing when they are 'changing' the numerator – they are creating equivalent fractions because the denominator has changed.
If necessary, show some of the fractions separately on the chalkboard.
For example:

$$\frac{2}{5x} = \frac{?}{15x^3}$$

Ask: *What has the original denominator been multiplied by to get the new denominator?*
Say: *We need to multiply the original numerator by the same number to make an equivalent fraction.*

5. Say: *These should have been straightforward examples. We are now going to do a few examples more relevant to Grade 10. With these examples, there will be expressions involving more than one term in the denominator's position. The only way to find the lowest common denominator will be to factorise the denominators first.*

6. Write the following examples on the chalkboard.
   Learners should write them in their books, making notes as they do so.

$$\frac{2}{x+5} + \frac{3}{x-3}$$

Say: *Look at the denominators.*
Ask: *Are the denominators in their simplest form or can they be factorised?*
(They are already in their simplest form, so they cannot be factorised).
Say: *We need to find the lowest common denominator.*
Ask: *can you see that the two expressions do not have a common factor?*
Note: If any learners say that x is common to both expressions, ask what x can be multiplied by to get $x + 5$ and repeat the question for $x – 3$.
Point out that as there is nothing that can be multiplied by x to get back to the expression then it cannot be a common factor.
Ask: *What is the lowest common denominator?*
$(x + 5)(x – 3)$
If any learners struggle with this idea, ask:
What is the LCD of 4 and 5?
(It is 20 which is made up of (4)(5)).

$$\frac{2}{x+5} + \frac{3}{x-3}$$

$$= \frac{2(x-3)}{(x+5)(x-3)} + \frac{3(x+5)}{(x+5)(x-3)}$$

$$= \frac{2(x-3)+3(x+5)}{(x+5)(x-3)}$$

$$= \frac{2x-6+3x+15}{(x+5)(x-3)}$$

$$= \frac{5x+9}{(x+5)(x-3)}$$

$$\frac{2}{x+2} - \frac{1}{x^2-4}$$

Tell learners to look at the denominators.

Ask: *Are the denominators in their simplest form or can they be factorised?*

(Yes they can be factorised: $x^2 - 4$ is a difference of two squares).

Once this has been factorised, say: We need to find the lowest common denominator.

Ask: *What is the smallest expression that BOTH denominators can go INTO?*

$(x + 2)(x - 2)$

$$\frac{2}{x+2} - \frac{1}{x^2-4}$$

$$= \frac{2}{x+2} - \frac{1}{(x+2)(x-2)}$$

$$= \frac{2(x-2)}{(x+2)(x-2)} - \frac{1}{(x+2)(x-2)}$$

$$= \frac{2(x-2)-1}{(x+2)(x-2)}$$

$$= \frac{(2x-4-1)}{(x+2)(x-2)}$$

$$= \frac{2x-5}{(x+2)(x-2)}$$

$$\frac{3}{x^2+6x+9} - \frac{2}{(x^2-9)} - \frac{1}{(x^2-6x+9)}$$

Tell learners to look at the denominators.

Ask: *Are the denominators in their simplest form or can they be factorised?*

(Yes the denominators can be factorised:

$x^2 + 6x + 9$ is a trinomial and will have 2 factors

$x^2 - 9$ is a difference of two squares

$x^2 + 6x + 9$ is a trinomial and will have 2 factors).

Once these have been factorised, say: We need to find the lowest common denominator.

Ask: *What is the smallest expression that BOTH denominators can go INTO?*

$(x + 3)(x + 3)(x - 3)(x - 3)$

$$\frac{3}{x^2+6x+9} - \frac{2}{x^2-9} - \frac{1}{x^2-6x+9}$$

$$= \frac{3}{(x+3)(x+3)} - \frac{2}{(x+3)(x-3)} - \frac{1}{(x-3)(x-3)}$$

$$= \frac{3(x-3)^2}{(x+3)^2(x-3)^2} - \frac{2(x+3)(x-3)}{(x+3)^2(x-3)^2} - \frac{1(x+3)^2}{(x+3)^2(x-3)^2}$$

$$= \frac{3(x-3)^2 - 2(x+3)(x-3) - 1(x+3)2}{(x+3)^2(x-3)^2}$$

$$= \frac{3(x^2-6x+9) - 2(x^2-9) - 1(x^2+6x+9)}{(x+3)^2(x-3)^2}$$

$$= \frac{3x^2-18x+27-2x^2-18-x^2-6x-9}{(x+3)^2(x-3)^2}$$

$$= \frac{24x}{(x+3)^2(x-3)^2}$$

$$\frac{3-x}{2x^2-x-3} - \frac{2}{6x-9} + \frac{4}{5x+5}$$

Tell learners to look at the denominators.

Ask: *Are the denominators in their simplest form or can they be factorised?*

(Yes the denominators can be factorised:

$2x^2 - x - 3$ is a trinomial and will have 2 factors

$6x - 9$ has a common factor

$5x + 5$ has a common factor).

Once these have been factorised, say:

We need to find the lowest common denominator.

Ask: *What is the smallest expression that BOTH denominators can go INTO?*

$15(2x - 3)(x + 1)$

$$\frac{3-x}{2x^2-x-3} - \frac{2}{6x-9} + \frac{4}{5x+5}$$

$$= \frac{3-x}{(2x-3)(x+1)} - \frac{2}{3(2x-3)} + \frac{4}{5(x+1)}$$

$$= \frac{15(3-x)}{15(2x-3)(x+1)} - \frac{5(2)(x+1)}{15(2x-3)(x+1)} + \frac{3(4)(2x-3)}{15(2x-3)(x+1)}$$

$$= \frac{45-15x}{15(2x-3)(x+1)} - \frac{10x+10}{15(2x-3)(x+1)} + \frac{24x-36}{15(2x-3)(x+1)}$$

$$= \frac{45-15x-(10x+10)+24x-36}{15(2x-3)(x+1)}$$

$$= \frac{45-15x-10x-10+24x-36}{15(2x-3)(x+1)}$$

$$= \frac{-x-1}{15(2x-3)(x+1)}$$

7. Ask learners if they have any questions.

8. Summarise the basic steps to follow with learners and tell them to write them down in their books.

Addition and Subtraction of Algebraic fractions:

- Factorise all denominators
- Find the LCD (Lowest common denominator)
- Adjust the numerators accordingly to create equivalent fractions
- Use the distributive law to remove any brackets
- Collect like terms in the numerator

9. Ask directed questions to ascertain learners' level of understanding.

10. Give learners an exercise to complete on their own.

11. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=jgGBdTL-OUw

https://www.youtube.com/watch?v=Z-42CAIljoI

TERM 1, TOPIC 1, LESSON 10

REVISION AND CONSOLIDATION

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners will have revised: ● all concepts covered in this topic.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up two or three of the first questions.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	36 38	Rev	30	w/sh	43	1.26 1.27	36 38	1.11	36

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Ask learners questions which will help them to revise what they have learned in this section. Point out issues that you know are important as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

1. Tell learners that before they do a revision exercise on algebra, you are going to work through a past examination paper question with them.

> Do not stick directly to the teaching notes. Include whatever assistance your learners need. Try to get as much information from them as possible by asking many questions. Some concepts may need to be taught again.

2. Learners should take it down into their books.

1. Factorise the following expressions fully:
   a) $x^2 - x$
   b) $3x^2 + 3px - 2mx - 2mp$
   c) $2p^2 - 2p - 12$
2. Simplify the following:

$$\frac{x^2 - x - 1}{x^3 + 1} \div \frac{2x}{2x + 2}$$

NSC NOV 2016

Teaching notes:

1. Remind learners that for all factorising questions, the starting point should ALWAYS be – how many terms are there and is there a common factor?
   a) Ask: *How many terms?*
      (2)
      Is there a common factor?
      (Yes - x).

b) Ask: *How many terms?*
(4)
Is there a common factor?
(No)
Ask: *What can we look for when there are four terms?*
(Grouping).

c) Ask: *How many terms?*
(3)
Is there a common factor?
(No).
Ask: *What can we look for when there are three terms?*
(Trinomial).

2. Ask: *What is the rule for division of fractions?*
(Change to multiplication and reciprocate the fraction after the change).
Ask: *What will the next step be?*
(Factorise and simplify)

Solutions:

1.

a) $x^2 - x$

$= x(x - 1)$

b) $3x^2 + 3px - 2mx - 2mp$

$= 3x(x + p) - 2m(x + p)$

$= (x + p)(3x - 2m)$

c) $2p^2 - 2p - 12$

$= 2(p^2 - p - 6)$

$= 2(p - 3)(p + 2)$

2. $\dfrac{x^2 - x - 1}{x^3 + 1} \div \dfrac{2x}{2x + 2}$

$= \dfrac{x^2 - x - 1}{x^3 + 1} \times \dfrac{2x + 2}{2x}$

$= \dfrac{x^2 - x - 1}{(x + 1)(x^2 - x + 1)} \times \dfrac{2(x + 1)}{2x}$

$= \dfrac{1}{x}$

1. The value of $\sqrt{33}$ lies between two integers. Find these integers without finding the exact value of $\sqrt{33}$.
2. Simplify: $\left(\frac{1}{p} - q\right)\left(\frac{1}{p} + q\right) - \frac{q}{p^2}\left(\frac{1}{q} + qp^2\right)$
3. Factorise completely:
   a) $6p + 40 - p^2$
   b) $-xy - (y - x)b + b^2$

Gauteng JUNE 2018

Teaching notes:

1. Say: *Remind me how we go about finding the integers.*
   (Find the perfect squares on either side of the given number).
2. Discuss how to approach this question.
   Ask: *What do you notice about the first term?*
   (It is a difference of two squares).
   Point out that this creates a shortcut which will be useful, particularly when working with fractions. To multiply the first and last terms will save time and assist in avoiding careless errors. When dealing with the second term, because fractions are involved, advise learners to put qp^2 over 1 to be clear where the denominators and numerators are.
3. a) Ask: *How many terms?*
   (3)
   Is there a common factor?
   (No)
   Ask: *What can we look for when there are three terms?*
   (Trinomial).
   Say: *This trinomial is not in the order we are used to seeing. This means we need to re-order it. Remember that the sign to the left of a term belongs to it.*
   Once this has been done, say:
   We can't factorise this until we have taken out a common factor of –1.

   b) Ask: *How many terms?*
   (3)
   Is there a common factor?
   (No)
   Ask: *What can we look for when there are 3 terms?*
   (Trinomial)
   Say: *It is clear that this is not a normal trinomial and certainly doesn't look like it can be factorised in the current format.*
   Ask: *What do you think can be done?*
   (Multiply out and create 4 terms then check for grouping).

Solutions:

1. $\sqrt{25} < \sqrt{33} < \sqrt{36}$

   $5 < \sqrt{33} < 6$

   $\therefore \sqrt{33}$ lies between 5 and 6

2. $\left(\frac{1}{p} - q\right)\left(\frac{1}{p} + q\right) - \frac{q}{p^2}\left(\frac{1}{q} + qp^2\right)$

   $= \left(\frac{1}{p^2} - q^2\right) - \frac{1}{p^2} - q^2$

   $= \frac{1}{p^2} - q^2 - \frac{1}{p^2} - q^2$

   $= 2q^2$

3. a) $6p + 40 - p^2$
   $= -(p^2 - 6p - 40)$
   $= -(p + 4)(p - 10)$

   b) $-xy - (y - x)b + b^2$
   $= -xy - by + bx + b$^2
   $= -y(x + b) + b(x + b)$
   $= (x + b)(-y + b)$
   $= (x + b)(b - y)$

3. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

4. Give learners an exercise to complete on their own.

5. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://learn.mindset.co.za/resources/mathematics/grade-10/term-1-revision/learn-xtra-live-2013/revision-algebraic-expressions-exponents

(Revision of algebraic expressions and exponents)

Term 1, Topic 2: Topic Overview

EXPONENTS

A. TOPIC OVERVIEW

A

- This topic is the second of five topics in Term 1.
- This topic runs for two weeks (9 hours).
- It is presented over five lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Exponents is part of Algebraic expressions which counts 30% of the final Paper 1 examination.
- Exponents is part of Algebra which forms the foundation for all topics in Mathematics. It prepares learners for both Calculus and Statistics.

Breakdown of topic into 5 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	Simplifying using exponential laws and definitions	2	4	Exponential equations	2
2	Simplifying using prime factors and factorisation	2	5	Revision and Consolidation	2
3	Rational exponents	1			

B

SEQUENTIAL TABLE

Senior phase	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Four laws and two definitions of exponents • Simplify expressions using the laws of exponents	• Four laws and two definitions of exponents • Simplify expressions using the laws of exponents • Solve equations using the laws of exponents • Accept that the rules also hold for exponents being rational numbers	• Apply the laws of exponents to expressions involving rational exponents • Add, subtract, multiply and divide simple surds • Demonstrate an understanding of the definition of a logarithm • Solve real life problems involving exponents and logarithms.

C

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Exponents.

The report makes the following suggestions:

- Exponential laws should be fully revised and revisited throughout a learner's high school career. More challenging examples on application of exponential laws should be practised regularly. Algebraic skills need to be improved, especially in Grade 10 and 11.
- Learners should be provided with additional tasks to enable them to become proficient in using factorisation when simplifying exponents.

It is important that you keep these issues in mind when teaching this section.

ASSESSMENT OF THE TOPIC

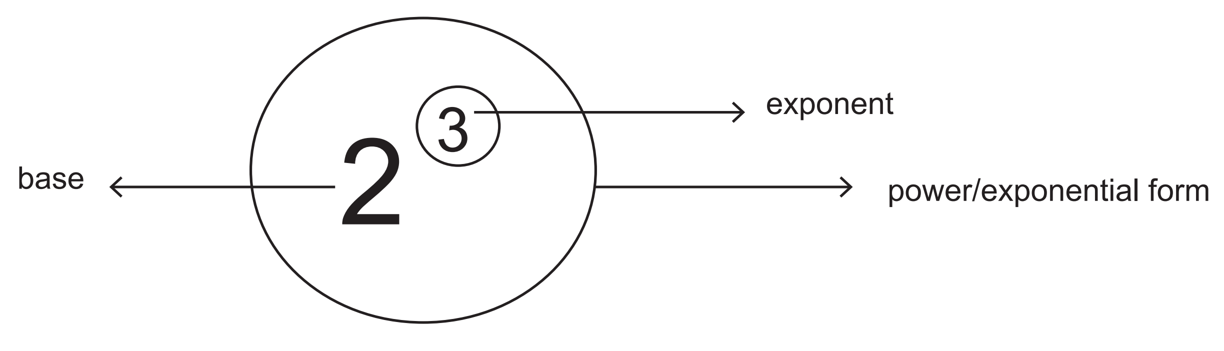
D

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test with memorandum (Resource 7); and an investigation with a rubric (Resource 6) are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of simplifying expressions using the laws of exponents.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

<table>
<tr><th>Term</th><th>Explanation</th></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>exponent (or index)</td><td>The superscript digit that is written above the base and indicates the number of times the base is repeated in a multiplication calculation.
An exponent shows how many times a constant or variable is a factor.
It represents the number of times the base is used as a factor.</td></tr>
<tr><td>Power (or exponential form)</td><td>This represents the number arrived at by raising a base to an exponent. It is ther product of repeated multiplication</td></tr>
<tr><td>base</td><td>The base is the number multiplied by itself</td></tr>
<tr><td>prime number</td><td>A number with only two factors, 1 and itself</td></tr>
</table>

composite number	A number with more than two factors
prime factors	The factors that make up the number that are prime numbers These numbers only have 1 and themselves as factors
rational number	Number that can be written in the form a/b where $a,b \in$ Z and $b \neq 0$)

SIMPLIFYING USING EXPONENTIAL LAWS AND DEFINITIONS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • simplify expressions using the laws (rules) and definitions of exponents.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the table ready to populate during the discussion.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 2 3	40 41 42	20 – 27	32 – 38			2.1 2.2 2.3	40 42 44	2.1	49

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Although learners have used an understanding of exponents in the previous section, it is important that they make exponents their focus now and practice all the laws.
2. This topic tends to cause difficulty for many learners. Ensure that quality time is spent on exponents to ensure confidence in learners.

DIRECT INSTRUCTION

1. Ask learners what rules they remember from exponents and to explain them with an example. Populate the table as you go, repeating good explanations and topping up with extra information as necessary.
2. Ensure the summary has a minimum of the following in it and that learners write it in their books:

	Law	Example	Explanation
1	$x^a \times x^b = x^{a+b}$	$2^3 \times 2^2 \times 2$ $= 2^{3+2+1}$ $= 2^6$	When multiplying powers with like bases keep the bases the same and add the exponents.
2	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{6x^6}{2x^2} = 3x^4$	When dividing powers with like bases keep the base and subtract the exponent. Divide numbers as per normal.
3	$(x^a)^b = x^{ab}$	$(-2a^2b^3)^2$ $= (-2)^2 \times a^{2\times 2} \times b^{3\times 2}$ $=4a^4b^6$	When raising a power to another power, keep the base and multiply the exponents.
4	$(xy)^a = x^a y^a$	$(a^4b)^3$ $= a^{12}b^3$	When more than one base is raised to an exponent, each base is raised to the exponent.
	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$\left(\frac{a^3}{b}\right)^3 = \frac{a^9}{b^3}$	When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.

3. Remind learners that these four basic laws are basically a shortcut when simplifying expressions with exponents.

> If the learners are unsure of a law, encourage them to write the question out in full. This often helps them understand where the answer came from .
> Example: $(3a^2b^3)^4 = (3a^2b^3) \times (3a^2b^3) \times (3a^2b^3) \times (3a^2b^3)$
> $= (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b)$
> Now all the same bases can be counted and brought together in exponential form:
> $= 3^4 a^8 b^{12} = 81a^8b^{12}$
> This is not recommended all the time – as you can see it is quite a long process, but it does help to show why the laws apply as they do.

4. Discuss the two definitions of exponents:

Zero-exponent definition	$x^0 = 1$	$(x^4 + 4)^0 + 3^0$ $= 1 + 1$ $= 2$	Any base raised to the power of zero is equal to 1. ($x \neq 0$ as 0^0 is undefined)
Negative-exponent definition	$x^{-a} = \frac{1}{x^{a}}$	$3x^{-2} = \frac{3}{x^2}$ and $\frac{3}{x^{-2}} = 3x^2$	A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.

Discuss and explain these definitions further as they are still relatively new to learners.

5. Say: *Answers with exponents should never have negative exponents as these can be simplified further. Let's look at why the negative exponent definition from above works as it does:*
Say: *Law 2 will help show this. Write the following example in your books:*

Simplify: $\frac{x^2}{x^5}$

Show learners the following two methods to demonstrate (explain that working across uses the longer method of expanding and working down uses Law 2):

$$\frac{x^2}{x^5} = \frac{x.x}{x.x.x.x.x} = \frac{1}{x^3} \quad \text{(after simplifying the } x\text{'s)}$$

OR $= x^{2-5}$

$= x^{-3}$

Tell learners that this demonstration should prove that: $\boxed{x^{-3} = \frac{1}{x^3}}$

6. Consider a more complex example:

Simplify:	Note: Some bases have positive exponents and others have negative exponents.
$\dfrac{x^5.y^{-3}.z^{-1}}{x^{-2}.y^2}$ $=\dfrac{x^5.x^2}{y^2.y^3.z^1}$ $=\dfrac{x^7}{y^5z}$	The bases with positive exponents should remain where they are. Deal with the bases with negative exponents – each base with a negative exponent needs to be 'moved' to the other side of the fraction to make the exponent positive. Once all bases are in the correct place (and all the exponents are positive), use Law 1 and Law 2 to simplify where possible.

7. Say: Answers with exponents should also not have zero as an exponent as it can be simplified further. Let's look at why the zero-exponent definition from above works as it does:
   Say: Law 2 will also help show this. Write the following example in your books:

   Simplify: $\dfrac{x^5}{x^5}$

   Show the following two methods to demonstrate (explain that working across uses the longer method of expanding and working down uses Law 2):

$$\frac{x^5}{x^5} = \frac{x.x.x.x.x}{x.x.x.x.x} = 1$$

OR $= x^{5-5}$

$= x^0$

This demonstration should prove that $\boxed{x^0 = 1}$

> Another way to help learners remember the definition about a zero exponent is to say that *'power zero means that there are none of those bases left after simplifying has taken place'*.
> So y^0 means that all y's must have been can cancelled out due to the simplification process and now there are no y's left at all.

8. Ask if there are any questions.
9. Say: *Let's look at some common misconceptions that we need to avoid.*

> Tell learners to write these in their books under a large heading that makes it clear that the following are NOT true.

Incorrect answer (misconception)	**Explanation of misconception**	**Correct answer**
$3^2 = 6$	Learners are multiplying the base with the exponent instead of using the exponent to show how many times to multiply the base with itself	$3^2 = 3 \times 3 = 9$
$-2 = \frac{1}{2}$	Once learners have heard of/learnt the negative exponent definition, they think anything negative can be made positive by 'moving it to the other side of the fraction'. Use a number line to show that $-2 \neq \frac{1}{2}$	$-2 = -2$
$\frac{1}{2x^{-3}} = 2x^3$	Learners use the negative exponent definition on a base that does not have a negative exponent.	$\frac{1}{2x^{-3}} = \frac{x^3}{2}$

Do some fully worked examples using these laws and definitions.
Learners should write the examples in their books and make notes as they do so.

Examples Simplify:	Teaching notes:
$a^3(a^2)^4$	Ask: *What should we deal with first?* (Brackets). Ask: *What law should we use when removing the brackets?* (When a power is raised to another power, multiply the exponents). Note: Complete to this step then ask: *What law should be used next?* (When powers of the same base are multiplied, add the exponents).
$a^3(a^2)^4$ $= a^3.a^8$ $= a^{11}$	

$\dfrac{x^2y^4 \times x^3y^5}{(x^4y^3)^2}$	Ask: *What should we deal with first?* (Brackets). Ask: *What law should we use?* (When a power is raised to another power, multiply the exponents). Once this step has been completed, this question could be approached in a few different ways (and in fact the numerator could have been simplified at the same time the brackets in the denominator were dealt with). Ask learners what to do next and if it is mathematically correct, follow their lead. In this example, the multiplication of powers of the same base by adding the exponents was done first, but simplifying could have taken place prior to this step due to multiplication and division.
$\dfrac{x^2y^4 \times x^3y^5}{(x^4y^3)^2}$ $= \dfrac{x^2y^4 \times x^3y^5}{x^8y^6}$ $= \dfrac{x^5y^9}{x^8y^6}$ $= x^{-3}y^3$ $= \dfrac{y^3}{x^3}$	
$\dfrac{12a^3b^{-2}}{8ab}$	Ask: *What should we deal with first?* (Simplifying the numerical digits and dealing with the negative exponent). Once this has been completed, ask: *What rule(s) can we use to simplify further?* (Multiplying powers of the same base and adding exponents as well as dividing powers of the same base and subtracting the exponents).
$\dfrac{12a^3b^{-2}}{8ab}$ $= \dfrac{3a^3}{2abb^2}$ $= \dfrac{3a^3}{2ab^3}$ $= \dfrac{3a^2}{2b^3}$	

$\frac{(abc^2)^4}{a^2b^{-3}c^3} \div \frac{a^2b^{-1}c^3}{(a^{-4}b^2c^4)^0}$	Ask: *What should we deal with first?* (Change division to multiplication and reciprocate the fraction after the change). Once this has been completed, ask: *How can we simplify further?* (Raise a power to another power by multiplying the exponents; use the zero-exponent definition to make the second numerator equal to 1). Once this has been completed, ask: *What can we do next?* (Use the negative-exponent definition to make all exponents positive). Once this has been completed, ask: *What law(s) can we use to simplify further?* (Multiplying powers of the same base and adding exponents as well as dividing powers of the same base and subtracting the exponents).
$\frac{(abc^2)^4}{a^2b^{-3}c^3} \div \frac{a^2b^{-1}c^3}{(a^{-4}b^2c^4)^0}$ $= \frac{(abc^2)^4}{a^2b^{-3}c^3} \times \frac{(a^{-4}b^2c^4)^0}{a^2b^{-1}c^3})$ $= \frac{a^4b^4c^8}{a^2b^{-3}c^3} \times \frac{1}{a^2b^{-1}c^3}$ $= \frac{a^4b^7c^8}{a^2c^3} \times \frac{b}{a^2c^3}$ $= \frac{a^4b^8c^8}{a^4c^6}$ $= b^8c^2$	

10. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

11. Give learners an exercise to complete with a partner.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wsaH5CARIHI

https://www.youtube.com/watch?v=X72qoK6i2B8

https://www.youtube.com/watch?v=gsYEe-JxS5k

https://www.youtube.com/watch?v=QX9GV8TGuII

TERM 1, TOPIC 2, LESSON 2

SIMPLIFYING USING PRIME FACTORS AND FACTORISATION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	21

Lesson Objectives

By the end of the lesson, learners should be able to:

- simplify expressions by using prime factors and the laws of exponents
- simplify expressions by using factorisation and the laws of exponents.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	45			5.1 5.2	46 48	2.5 2.6	48 50		

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This topic extends learner's knowledge regarding laws of exponents; as well as factorising skills.
2. There is almost always at least one question like those in this topic in every Grade 10 and Grade 11 assessment that includes exponents.
3. Ensure learners are confident in the laws and types of examples from the previous lesson before continuing with this lesson.

DIRECT INSTRUCTION

1. Tell learners that a good knowledge of the laws of exponents as well as factorising are skills required for the types of questions to be covered today.
2. The lesson will be made up of four fully worked examples – two of each type. If, at the end of the examples, you feel your learners need more examples before trying an exercise on their own, source a few more examples.
3. Before starting the examples, ask learners:

What is a prime number?	A number that has only two factors – 1 and itself.
What is a composite number?	A number that has more than two factors
Give an example of a composite number written as a product of its prime factors (start with numbers less than 20)	A few possibilities: $10 = 2 \times 5$ $15 = 3 \times 5$ $16 = 2 \times 2 \times 2 \times 2$ $18 = 2 \times 3 \times 3$
How do we write large composite numbers as a product of their prime factors?	Use the ladder method and divide prime numbers into the composite number until you get 1. (Do an example with learners) $\begin{array}{r\|l} 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$ $\therefore 70 = 2 \times 5 \times 7$

4. Say: *The skills that we have just practiced (writing a number as a product of prime factors) will be needed for this topic.*

5. Tell learners that you will work through two types of exponent questions.
   Learners should write the examples in their books and make notes as they do so.

Examples of Type 1 questions	
$\dfrac{3^{n+1}.4^{n-1}}{2^{n}.6^{n-1}}$	
Point out to learners that there is only one term in both the numerator and the denominator. In this case, each base must be written as a product of prime factors, then the laws and definitions of exponents can be used to simplify. Ask: *Which bases can still be broken down into a product of prime factors?* (4 and 6) Write these as products of their prime factors. ($4 = 2^2$ and $6 = 2 \times 3$)	
$\dfrac{3^{n+1}.4^{n-1}}{2^{n}.6^{n-1}}$ $= \dfrac{3^{n+1}.(2^{2})^{n-1}}{2^{n}.(2.3)^{n-1}}$	Say: *The first step is to write all composite numbers as a product of their prime factors.* Once this has been done, ask: *What should we deal with next?* (Brackets)
$= \dfrac{3^{n+1}.2^{2n-2}}{2^{n}.2^{n-1}.3^{n-1}}$	Ask: *What law(s) should be used?* (Raising a power to another power and multiplying and when more than one base is raised to an exponent, each base is raised to that exponent) Once this has been done, ask: *What can be done next?* (Use the law to collect like bases together by adding the exponents).
$= \dfrac{3^{n+1}.2^{2n-2}}{2^{2n-1}.3^{n-1}}$	Once this has been done, ask: *What can be done next?* (Use the law to divide powers of the same base by subtracting exponents).

$= 3^{n+1-(n-1)}.2^{2n-2-(2n-1)}$ $= 3^{n+1-n+1}.2^{2n-2-2n+1}$	Remind learners to use brackets when there are two terms in the expression being subtracted. The brackets show what is being subtracted. It is good practice to get into the habit of always using brackets to ensure that errors are not made. Point out that once the subtraction is shown, the distributive law should be used and like terms collected.
$= 3^{2}.2^{-1}$ $= \frac{9}{2}$	Once this has been done, ask: *Can we simplify further?* (Yes – deal with the negative exponent and $3^2 = 9$)
$\frac{6^{2x}.11^{2x}}{22^{2x-1}.3^{2x}}$	
Point out again that there is only one term in both the numerator and denominator. Ask: *Which bases can still be broken down into a product of prime factors?* (6 and 22) *Write these bases as products of their prime factors.* (6 = 2 × 3 and 22 = 2 × 11)	
$\frac{6^{2x}.11^{2x}}{22^{2x-1}.3^{2x}}$ $= \frac{(2.3)^{2x}.11^{2x}}{(2.11)^{2x-1}.3^{2x}}$	Composite numbers written as prime factors.
$= \frac{2^{2x}.3^{2x}.11^{2x}}{2^{2x-1}.11^{2x-1}.3^{2x}}$	Removie brackets by applying the power raised to another power law and more than one base raised to an exponent law.
$= 2^{2x-(2x-1)}.3^{2x-2x}.11^{2x-(2x-1)}$ $= 2^{2x-2x+1}.3^{2x-2x}.11^{2x-2x+1}$	Divide powers of the same base by subtracting exponents.
$= 2^{1}.3^{0}.11^{1}$	Simplify by collecting like terms within the exponents.
= 2 × 11 = 22	= 2 × 11 = 22

6. Before moving on to the second type of question, ask learners to try this one on their own using the two examples they have just done as a guide. Tell learners to take note where they begin struggling (if they do) so they can ask you specific questions at that point when you do it in full on the board for them.

$\frac{3^x 27^x + 1}{9^{2x+2}}$
$\frac{3^x 27^{x+1}}{9^{2x+2}}$ $= \frac{3^x (3^3)^{x+1}}{(3^2)^{2x+2}}$ $= \frac{3^x . 3^{3x+3}}{3^{4x+4}}$ $= \frac{3^{x+3x+3}}{3^{4x+4}}$ $= \frac{3^{4x+3}}{3^{4x+4}}$ $= 3^{4x+3-(4x+4)}$ $=3^{4x+3-4x-4}$ $=3^{3-4}$ $=3^{-1}$ $= \frac{1}{3^1} = \frac{1}{3}$

7. Tell learners that before moving on to the next type of question, you will discuss a skill that will be useful for the examples that follow.

8. Write $2^n . 2^1$ on the board. Ask learners to simplify it.
   Confirm they all used the law to add exponents when multiplying powers with like bases.
   $2^n . 2^1 = 2^{n+1}$

9. Say: *It is important to be able to work this in reverse so as to ensure that we factorise with accuracy.*
   Cover up the question (point 8) so that only the answer is visible – tell learners that if they see this, they need to be able to write what it must have looked like before the law was applied and the multiplication was made into one power only.

10. Write 5^{2x-1} on the board.
    Ask: *Write this in expanded form (what it looked like before simplifying).*
    Once learners have done so do it on the board to confirm they understand.

$$5^{2x-1} = 5^{2x} 5^{-1}$$

11. Do two fully worked examples of the second type of question.
    Learners should write the examples in their books and make notes as they do so.

<table>
<tr><td colspan="2">Examples of Type 2 questions</td></tr>
<tr><td colspan="2">$$\frac{3^{4x-1} - 3^{4x+1}}{3^{4x}}$$</td></tr>
<tr><td colspan="2">Point out that in this fraction there are two terms in the numerator.
The fact that there is more than one term should alert learners that factorisation will be essential in order to simplify. Remind them that they cannot simplify within a term that is next to a ‘+’ or ‘–‘ sign. Through factorising, one term will replace the two terms and simplification can then be done.
Tell learners that to find a common factor, and know what will be ‘left over’, we first need to ‘undo’ the expressions that have two term exponents as we have just practiced.</td></tr>
<tr><td>$\frac{3^{4x-1}3^{4x+1}}{3^{4x}}$
$= \frac{(3^{4x}.3^{-1}3^{4x}.3^{1}}{3^{4x}}$</td><td>Once this has been completed, ask:
How many terms are in the numerator?
(2).
Ask: Is there a common factor?
(Yes – 3^{4x}).
Say: We need to take out the common factor.</td></tr>
<tr><td>$= \frac{3^{4x}(3^{-1} - 3^{1})}{3^{4x}}$</td><td>Once this has been completed, ask:
What can be simplified now?
(3^{4x}).</td></tr>
<tr><td>$= 3^{-1} – 3^{1}$</td><td>Ask: Can this be simplified further?
(Yes – deal with the negative exponent then do the calculation).</td></tr>
<tr><td>$= \frac{1}{3} – 3$
$= \frac{-8}{3}$</td><td></td></tr>
<tr><td colspan="2">$$\frac{2.5^{x} + 5^{x-2}}{3.5^{x+1} - 7.5^{x-1}}$$</td></tr>
<tr><td colspan="2">Point out again that more than one term occurs within this fraction. In this case, both the numerator and denominator have two terms. The numerator and denominator both need to be factorised to create one term only so that simplification can take place.
Ask learners to try the first step on their own (re-writing the one base with two term exponents as two bases) before doing it on the board.</td></tr>
</table>

$\dfrac{2.5^{x}+5^{x-2}}{3.5^{x+1}7.5^{x-1}}$ $=\dfrac{2.5^{x}+5^{x}.5^{-2}}{(3.5^{x}.5^{1}-7.5^{x}.5^{-1}}$	Once this has been completed, ask: *How many terms are in the numerator?* (2). Ask: *Is there a common factor?* (Yes - 5^x). Repeat with the denominator. Point out that the common factor is the same in both the numerator and denominator – this is usually the case.
$=\dfrac{5^{x}(2+5^{-2})}{5^{x}(3.5-7.5^{-1})}$	Simplify
$=\dfrac{2+5^{-2}}{3.5-7.5^{-1}}$	Say: *You need to be very careful at this stage when dealing with negative exponents. As there are still two terms, you cannot simply 'move the base to the other side'. Negative exponents within the numerator or denominator need to be dealt with.*
$=\dfrac{2+\frac{1}{5^{2}}}{15-7.\left(\frac{1}{5}\right)}$	From this stage, learners need to be proficient in calculations with fractions.
$=\dfrac{2+\frac{1}{25}}{15-\frac{7}{5}}$ $=\dfrac{\frac{51}{25}}{\frac{68}{5}}$ $=\dfrac{51}{25}\div\dfrac{68}{5}$ $=\dfrac{51}{25}\times\dfrac{5}{68}$ $=\dfrac{51}{5}\times\dfrac{1}{68}$ $=\dfrac{51}{340}$	Tell learners that questions are not usually this complicated at the end but regular practice in working with fractions is excellent for their mathematics.

12. Before learners do an entire exercise on their own, ask them to try this one on their own, using the two examples they have just done as a guide. Tell them to take note where they begin struggling (if they do) so they can ask you specific questions at that point when you do it in full on the board for them.

$\dfrac{2.3^{n+2}}{3^n - 3^{n+1}}$
$= \dfrac{(2.3^n.3^2}{3^n - 3^n.3^1}$ $= \dfrac{2.3^n.3^2}{3^n(1 - 3^1)}$ $= \dfrac{2.3^2}{1 - 3^1}$ $= \dfrac{18}{-2}$ $= -9$

13. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

14. Give learners an exercise to complete on their own. If you use one of the two textbooks that don't have an exercise, source some questions from another textbook.

15. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=TQREh6JWzO0

TERM 1, TOPIC 2, LESSON 3

RATIONAL EXPONENTS

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • simplify expressions with rational exponents.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the first two examples.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
				5.3	49	2.4	46	2.2	51

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This is a new concept for learners. All exponents that they have encountered before have been integers.

DIRECT INSTRUCTION

1. Tell learners that all the rules that they have worked with for exponents can also be applied to rational numbers.
2. Ask: *What is a rational number?*
(A number that can be written in the form $\frac{a}{b}$ where $a,b \in$ Z and $b \neq 0$).
3. Do the following examples with learners.
Learners should write the examples in their books and make notes as they do so.

Examples Simplify:	Teaching notes:
$m^{\left(\frac{1}{2}\right)} \times (m^2)^{\frac{3}{4}}$ $= m^{\frac{1}{2}} \times m^{\frac{3}{2}}$ $= m^2$	Ask: *What laws should we apply to simplify?* (First, deal with the brackets – raising a power to another power the exponents must be multiplied; then we are multiplying powers of the same base so we need to add the exponents).
$(ab)^{\frac{4}{5}} \div a^{\frac{1}{5}} b^{\frac{3}{5}}$ $= a^{\frac{4}{5}} b^{\frac{4}{5}} \div a^{\frac{1}{5}} b^{\frac{3}{5}}$ $= a^{\frac{3}{5}} b^{\frac{1}{5}}$	Ask: *What laws should we apply to simplify?* (First, deal with the brackets– each base must be raised to the exponent; then we are dividing powers of the same base so we need to subtract the exponents).
Point out that although we need to work with fractions, the procedures to be followed are exactly the same as when the exponents are integers.	

4. Say: *When compound numbers are raised to a rational exponent, they can often be simplified by using prime factors and the rule for raising a power to another power.*
   For example:

Example	Teaching notes	Solution
$8^{\frac{1}{3}}$	For each one of these questions, every composite number needs to be written in exponential form using prime factors. Once that has been completed, the raising a power to another power law can be applied and if necessary, any other exponential rules to simplify further.	$8^{\frac{1}{3}}$ $= (2^3)^{\frac{1}{3}}$ $= 2$
$32^{\frac{3}{5}}$		$32^{\frac{3}{5}}$ $= (2^5)^{\frac{3}{5}}$ $= 2^3 = 8$
$\left(\frac{1}{81^{\frac{1}{4}}}\right)^3$		$\left(\frac{1}{81^{\frac{1}{4}}}\right)^3$ $= \left(\frac{1}{(3^4)^{\frac{1}{4}}}\right)^3$ $= \left(\frac{1}{3}\right)^3$ $= \frac{1}{27}$
$20^{\frac{1}{2}} \times 10^{\frac{1}{2}} \times 2^{\frac{1}{2}}$		$20^{\frac{1}{2}} \times 10^{\frac{1}{2}} \times 2^{\frac{1}{2}}$ $= (2^2.5)^{\frac{1}{2}} \times (2.5)^{\frac{1}{2}} \times 2^{\frac{1}{2}}$ $= 2.5^{\frac{1}{2}} \times 2^{\frac{1}{2}}.5^{\frac{1}{2}} \times 2^{\frac{1}{2}}$ $= 2^2.5$ $= 20$

5. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

6. Give learners an exercise to complete with a partner. If you use one of the textbooks without an exercise, source a few questions from another textbook.

7. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=zKE46XejGZU

https://www.youtube.com/watch?v=-T1punCdxas

https://www.youtube.com/watch?v=iPapJoyq0Gc

EXPONENTIAL EQUATIONS

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners should be able to: • solve exponential equations.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5 6 7	48 49 50	30 31 32	43 43 44			2.7 2.8	53 54	2.3	55

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have worked with basic exponential equations in Grade 9.
2. Revising these concepts is essential before moving on to higher level equations. Ensure that learners are confident with the basics from previous years before starting new work.

DIRECT INSTRUCTION

1. Point to the heading of the lesson on the chalkboard and ask: *What is an exponential equation?*
   (An equation in which the unknown is an exponent).
2. Say: *You worked with exponential equations last year. We need to revise those before moving up a level to Grade 10 concepts.*
   Do the following examples with learners, discussing them in detail as you do them.
   Learners should write the examples in their books and make notes as they do so.

Examples Solve for x:	Teaching notes:
$3^x = 3^4$ $\therefore x = 4$	Say: *The focus of solving an exponential equation is to make sure the bases are equal. If the bases are the same, the exponents must be the same.* Ask: *Can you see that this equation already has the same base on each side? It should therefore be easy to see what the unknown should be.*
$3^x = 27$ $3^x = 3^3$ $\therefore x = 3$	Ask: *Are the bases the same?* (No). Ask: *what can we do to make the bases the same?* (Write 27 as a product of its prime factors).

$2^x = \frac{1}{8}$ $2^x = \frac{1}{2^3}$ $2^x = 2^{-3}$ $\therefore x = -3$	Ask: *Are the bases the same?* (No) Ask: *What can we do to make the bases the same?* (Write 8 as a product of its prime factors) Ask: *Now can we say that the exponents must be equal?* (No) Ask: *What can be done?* (Use the negative exponent definition in reverse).
$5^x = 1$ $5^x = 5^0$ $\therefore x = 0$	Remind learners that we need the bases to be equal. Ask: *Is it possible to make the bases equal?* (Yes – we can use the zero-exponent definition).

Before doing the next example, discuss the following misconception with learners: When solving equations, learners often forget some basics they have been taught about exponents and the rules. Now that their focus is on equations, the following errors may be made: Possible error 1: $2.5^x = 10^x$ Point out that we cannot multiply unlike bases unless the exponents are equal $(2^x.3^x = 6^x)$ If 2.5^x is part of an equation, then learners can divide by 2 on both sides. Possible error 2: $\frac{2^x}{2} = 1^x$ Point out that we cannot simplify a base when it has an exponent in which the value is not known. The law of dividing powers of the same base and subtracting the exponents is required here.	
$3.2^{x+1} = 96$ $\frac{3.2^{x+1}}{3} = \frac{96}{3}$ $2^{x+1} = 32$ $2^{x+1} = 2^5$ $\therefore x + 1 = 5$ $x = 4$	Say: *We want the same base on each side. Before writing compound numbers as products of their prime factors we first need to deal with the 'multiply 3' on the left-hand side. To simplify, we can divide both sides by 3.* *Once this has been done, the procedure is the same as the previous examples.* Remind learners that with equations, the answers are easy to check: *Put the value found back into the left hand side of the equation and check that it is equal to 96.*

3. Ask learners if they have any questions.

If learners found any of these examples challenging, do a few more examples of a similar level before moving on.

Do the following examples.
Learners should write the examples in their books and make notes as they do so.

Examples Solve the following:	Teaching notes:
$2^x = 0{,}25$ $2^x = \frac{1}{4}$ $2^x = \frac{1}{2^2}$ $2^x = 2^{-2}$ $\therefore x = -2$	Say: *We need the bases to be equal when solving exponential equations.* Ask: *How can we make the bases equal?* (Change the decimal into a common fraction) From this point, learners should be able to tell you the steps that follow from previous examples.
$\frac{1}{3}.(3)^{x+1} = \frac{1}{9}$ $3.(3)^{x+1} = 1$ $3^{1+x+1} = 1$ $3^{2+x} = 1$ $3^{2+x} = 3^0$ $\therefore 2 + x = 0$ $\therefore x = -2$	Ask: *What can we do to simplify this equation?* (Multiply both sides by the lowest common denominator – 9). Ask: *What can we do to simplify further?* (Simplify the left-hand side of the equation using the law for multiplying powers of the same base by adding the exponents) Remind learners that we require equal bases. Ask: *How can we achieve this?* (Use the zero exponent definition to change 1 into a base 3).
$12\left(\frac{1}{4}\right)^x = 12\left(\frac{1}{4}\right)^2$ $\left(\frac{1}{4}\right)^x = \left(\frac{1}{4}\right)^2$ $\therefore x = 2$	This is simpler than it looks. The reason for using it as an example is to discuss with learners why they cannot simplify the denominator of 4 with the 12. Point out that the '4' is part of a power that has an exponent which means there won't only be one of them (unless the exponent is 1 which it is not). Ask: *What can we do to simplify this equation?* (Divide both sides by 12).

4. Ask if anyone has any questions before moving on to the final part of the lesson.

5. Mention to learners that although they don't deal with it yet, there is a method of finding an exponent if the bases are not the same. They will learn about this in Grade 12.

6. Tell learners that you are going to look at equations with rational exponents and the base is the unknown.
Do the following examples with learners.
Learners should write the examples in their books and make notes as they do so.

Examples Solve the following:	Teaching notes:
$x^{\frac{1}{2}} = 7$ $\left(x^{\frac{1}{2}}\right)^2 = 7^2$ $x = 49$	When solving for a base that has a rational exponent, raise both sides to the power of the denominator. Use 'raising a power to another power law' to simplify and get the base on its own. This can only be done if the base with the rational exponent is by itself. In the third example, both sides will be divided by 3 first to ensure this is the case.
$x^{\frac{1}{5}} = 2$ $x^{\frac{1}{5}\,5} = 2^5$ $x = 32$	
$3x^{\frac{1}{3}} = 12$ $\frac{3x^{\frac{1}{3}}}{3} = \frac{12}{3}$ $x^{\frac{1}{3}} = 4$ $\left(x^{\frac{1}{3}}\right)^3 = 4^3$ $x = 64$	

7. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

8. Give learners an exercise to complete on their own.

9. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=-OFC9iRyO1o

https://www.youtube.com/watch?v=q0a8a06AowY

TERM 1, TOPIC 2, LESSON 5

REVISION AND CONSOLIDATION

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	21
Lesson Objectives By the end of the lesson, learners will have revised: • simplifying expressions using the laws of exponents • solving exponential equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some ch	51 51	Rev	45			2.9 2.10	55 56	2.4	57

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Ask learners questions which will help them to revise what they have learned in this section. Point out issues that you know are important, as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

Ask learners to do the revision exercise from their textbook. If you have an extra worksheet or a past test paper, this would also be an excellent way for them to consolidate what they have learned. It would also give them the opportunity of knowing what to expect when they must do an assessment.

1. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
2. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 1, Topic 3: Topic Overview

NUMBER PATTERNS

A. TOPIC OVERVIEW

A

- This topic is the third of five topics in Term 1.
- This topic runs for one week (4,5 hours).
- It is presented over two lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 4,5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Number Patterns counts 15% of the final Paper 1 examination.
- Mathematics is especially useful when it helps you predict, and number patterns are all about prediction
- Working with number patterns leads directly to the concept of functions in mathematics: a formal description of the relationships among different quantities.
- Recognising number patterns is also an important problem-solving skill. If a pattern is recognised when looked at systematically, the pattern can be used to generalise what can be seen in a broader solution to a problem.

Breakdown of topic into 2 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Linear number patterns	2,5	2	Revision and Consolidation	2

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
Investigate and extend numeric and geometric patterns looking for relations ships between numbers, including patterns: • represented in physical or diagram form • not limited to sequences involving a constant difference or ratio • of learners' own creation • represented in tables • represented algebraically • describe and justify the general rules for relationships between numbers	Investigate number patterns leading to those where there is a constant difference between consecutive terms and the general term (without using a formula) is linear	• Quadratic patterns • Arithmetic and Geometric sequences • Arithmetic and geometric series, including sum to infinity • Sigma notation • Derivation of formulae for the sum of arithmetic and geometric series • Problem solving

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Number Patterns.

These include:

- Attention needs to be paid to the basics of Mathematics – this includes being able to substitute correctly and apply algebraic skills correctly
- The difference between position and value of a term must be emphasised
- Learners need to be exposed to many patterns – including those that include variables
- Learners should be able to establish their own patterns from diagrams or pictures.

It is important that you keep these issues in mind when teaching this section.

While teaching Number Patterns, always use the correct notation and mathematical language. Learners must be encouraged to do the same. A learner's understanding of the concepts is more important than merely doing routine procedures.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test, with memorandum, and an investigation, with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of recognising patterns and finding a term, a position of a term or the general formula.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
number pattern	List of numbers that follow a sequence or a pattern
consecutive	One after the other
linear pattern	Pattern formed by adding the same value every time (the value can be positive or negative)
common (or constant) difference	The value added each time to form a linear pattern
geometric pattern	Sequence of numbers with a constant ratio between consecutive terms
common ratio	The number used to multiply one term to get to the next term in a geometric sequence If division occurs, reciprocate and turn it into multiplication Example: $\div 5 = \times \frac{1}{5}$

rule **nth term** **general term**	Algebraic explanation of how a pattern is formed
term	Number in a given sequence. Example: In the sequence: 5 ; 0 ; –5 ; –10 , all four numbers represent terms and each one of them are in a particular position
position	The place in the sequence held by one of the terms. Example: In the sequence: 2 ; 4 ; 6 ; 8…., 6 is in the 3rd position

TERM 1, TOPIC 3, LESSON 1

LINEAR NUMBER PATTERNS

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	22

Lesson Objectives

By the end of the lesson, learners should be able to:

- recognise a linear pattern
- find the general term of a linear pattern
- find a term in a given position of a linear pattern
- find the position of a term in a linear pattern.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the three patterns from point 1 on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	57	1	49	6.1	55	3.1 3.2 3.3	59 61 64	3.1	64

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have encountered patterns throughout the Senior Phase. Ask learners questions to establish their existing knowledge.
2. Although an understanding of linear patterns is the only concept required by CAPS, this concept can be asked in many ways. It is therefore important to expose learners to as many types of patterns as possible to give them more practice and knowledge.

DIRECT INSTRUCTION

1. Ask learners to consider these three patterns:

   1 ;3 ;5 ;7….

   1 ;5 ;9 ;13….

   12 ;15 ;18 ;21…

2. Ask learners to copy the patterns and extend them by three more terms.
3. After a few minutes, ask three different learners to come to the board and write the next three terms down. Ask each learner how he/she knew what to do to get the next term.
4. Explanation for each pattern:
   add 2 each time; add 4 each time; add 3 each time
5. Ask: *What mathematical term do we use when the same number is added each time?* (Common difference).
6. Write these two patterns on the board and ask learners to copy them and extend the patterns by three terms:

   22 ;20 ;18 ;16….

   12 ;7 ;2 ; -3…

7. After a few minutes, ask two different learners to come to the board and write the next three terms down. Ask each learner how he/she knew what to do to get the next term.
8. Explanation for each pattern:
   Subtract 2 each time; subtract 5 each time

9. Point out that the subtraction of the same number each time is still dealing with a common difference. The same number is still being added each time – the number being added is a negative number.

10. Say: *All five patterns discussed so far are linear patterns. A linear pattern is a pattern formed by adding the same value every time (the value can be positive or negative).*

11. Remind learners of the linear graphs they drew in Grade 9. A straight-line graph has one gradient that ensures it is a perfect straight line. The common difference in the patterns discussed would ensure a straight line if the points were plotted.
The points used would be the term number (1; 2; 3 etc) with the terms themselves (1 ; 3 ; 5 ; 7 from pattern 1). The only difference between the line from a linear pattern and the line from a function would be that the linear pattern would form a line of discrete points while the linear function is more commonly drawn as a continuous line.

12. Learners should write the following definitions in their books.

linear pattern	A pattern formed by adding the same value every time (the value can be positive or negative).
common (or constant) difference	The value added each time to form a linear pattern.
term	A number in a given sequence. Example: In the sequence: 5 ; 0 ; -5 ; -10 , all four numbers represent terms and each one of them are in a particular position.
position	The place in the sequence held by one of the terms. Example: In the sequence: 2 ; 4 ; 6 ; 8…. 6 is in the 3rd position.

Note:
Discuss 'position' with learners. Point out that a position can only ever be positive and a natural number. It is not possible to have a term in a negative position or in a fractional position.
The diagnostic reports state that learners are often confused between a term and the position. Spend some time on these definitions and ask directed questions using the patterns on the board to ensure that learners understand the difference.
For example, ask,
In pattern 1, what term is in the 3rd position? (5)
In pattern 4, what is the position of the term 20? (2nd position)

Finding the rule to describe linear patterns

13. Confirm with learners that linear patterns have a constant difference. Tell them that this constant difference is essential to finding the rule of a pattern.

14. Say: *The general rule for a linear number pattern is:* $T_n = bn + c$
    Ask learners to write this in their books.
    Tell learners that the 'b' represents the constant difference. Tell them to label the 'b' and mark this in their books: 'b' → constant/common difference.

15. Tell learners you are going to do two examples on how to find the rule for a linear pattern. They need to copy these examples into their books.

Example:
Find the general term (rule) for the pattern: 9 ; 14 ; 19 ; 24…..
Teaching notes: Ask: *What is the common difference in this pattern?* (add 5) Ask: *Which term is in position 1?* (9) Say: *The common difference needs to be used to find the rule (or general term). Consider that 9 is the 1st term. Multiply the common difference by 1 (5×1)* Ask: *what still needs to be done to get to 9, which is the 1st term?* (still need to add 4) Tell learners to check by using term 2: 14 is in the 2nd position (it is term 2) 2 × 5 = 10. To get from 10 to 14 (which is term 2) 4 needs to be added. The rule is therefore: Multiply by 5 and add 4.
Solution: Common difference: +5 $T_n = bn + c$ $\therefore T_n = 5n + c$ Remind learners how the 'c' is found from the steps above $T_1 = 5(1) + c$ $9 = 5 + c$ $4 = c$ $\therefore T_n = 5n + 4$
Note: Point out to learners that this rule can be used to answer many questions such as, which term is in position 20 or in which position is a certain term. Tell learners that we will look at these types of questions after doing one more example on how to find the rule when given a pattern.

Example: Find the general term (rule) for the pattern: 22 ; 19 ; 16 ; 13…..
Teaching notes: Ask: *What is the common difference in this pattern?* (subtract 3) Ask: *Which term is in position 1?* (22) Say: *Use this and the common difference to find what must be done to get 22.*
Solution: Common difference: –3 $T_n = bn + c$ $\therefore\ T_n = -3n + c$ Remind learners how the 'c' is found from the steps above (-3×1=-3. To get to 22 from -3, we need to add 25) $T_1 = -3(1) + c$ $22 = -3(1) + c$ $25 = c$ $\therefore\ T_n = -3n + 25$

16. Give the following two patterns to learners and ask them to find the rule:

4 ;7 ;10 ;13…

16 ;9 ;2 ; -5

Solutions: $T_n = 3n + 1$ and $T_n = -7n + 23$

17. Point out the following to learners:
    T_n means the n^{th} term, in other words the general term or the rule.
    The n represents the position of a term.
    For example, T_3 means Term number 3 (position 3)

18. Ask learners if they have any questions before moving on to using the rules to answer questions.

19. Using the rule $T_n = 3n + 1$ (found above), ask the following question:
    What term would be in position 12?

20. Ask: *Which part of the rule represents the position of a term?*
    (n)
    Say: *If we need to find which term is in a certain position, we need to substitute the position given into n's place and find the term concerned.*
    Note that this requires the algebraic skill, substitution.

21. Do the question on the board for learners now and ask them to copy it into their books.

$$T_n = 3n + 1$$

$$T_{12} = 3(12) + 1$$

$$T_{12} = 37$$

37 is the term in the 12th position in the sequence 4 ; 7 ; 10 ; 13…

22. Say: *Use the same rule to find the terms in position 10 and position 50.*
    Solutions: $T_{10} = 31$ and $T_{50} = 151$

23. Ask: *Using the same rule* ($T_n = 3n + 1$)*, in which position is the term 64?*

24. Ask: *What has been given and what is required?*
    (We are given the term and require the position).
    Ask: *What part of the rule (general term) is unknown?*
    (the n).
    Say: *If we need to find which position a certain term is in, we need to substitute the term given into T_n's place and find the term value of n.*
    Note that this requires the algebraic skill: solving an equation.

25. Do the question on the board for learners.
    Learners should write the answer in their books.

$$T_n = 3n + 1$$

$$64 = 3n + 1$$

$$63 = 3n$$

$$21 = n$$

The term 64 is in position 21 in the sequence 4 ; 7 ; 10 ; 13…

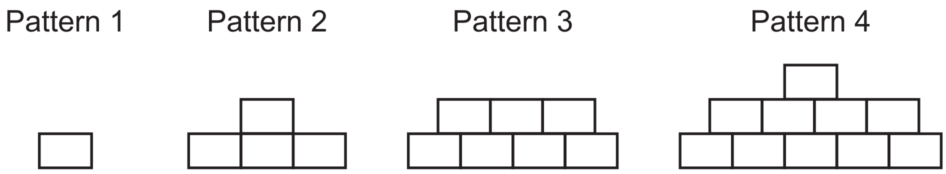
26. Say: *Use the same rule to find the position of terms 46 and 121 in the sequence.*
    Solutions: 15th and 40th positions.

27. Ask learners if they have any questions.

28. Do further examples with learners. They should copy them into their books.

Example:

Consider the following pattern formed by bricks being laid:



(i) List the number of bricks in each of the patterns shown above.
(ii) How many bricks will be in the next pattern?
(iii) Find the general term which represents this pattern.
(iv) Use the rule to find how many bricks will be required for the 12^{th} pattern.
(v) Which pattern will need 52 bricks?

Teaching notes:

Tell learners that pattern questions with the use of diagrams are commonly asked.
Learners may have also had some in the exercise from their textbook done earlier.
(i) and (ii) should be easy for learners to answer.

(iii)

By now learners should find this straightforward.

(iv)

Remind learners that, when given the position, substitution is required.

(v)

Remind learners that, when given the term, solving equations is required (solve for n).

Solution:

(i) 1 ; 4 ; 7 ; 10

(ii) 13

(iii) $T_n = 3n - 2$

(iv) 34

(v) $52 = 3n - 2 \therefore n = 18$

Example:

Consider the sequence: $\frac{2}{3}$; $\frac{5}{8}$; $\frac{8}{13}$; $\frac{11}{18}$ ….

(i) Determine the n^{th} term

(ii) Find the 10^{th} term.

Teaching notes:

Learners may find this question difficult. Ask learners to look at all the numerators on their own and at all the denominators on their own. Tell them to consider the pattern in the numerators and the pattern in the denominator separately.
Each pattern is a linear sequence.

Solutions:

(i) 2 ; 5 ; 8 ; 11…

Common difference in numerator sequence: 3

$$T_n = bn + c$$
$$\therefore\ T_n = 3n + c$$

Use term 1 to find c:

$$T_1 = 3(1) + c$$
$$2 = 3(1) + c$$
$$-1 = c$$

$\therefore\ T_n = 3n - 1$

3 ;8 ;13 ;18…

Common difference in denominator sequence: 5

$$T_n = bn + c$$
$$\therefore\ T_n = 5n + c$$

Use term 1 to find c:

$$T_1 = 5(1) + c$$
$$3 = 5(1) + c$$
$$-2 = c$$

$\therefore\ T_n = 5n - 2$

Final solution: $\frac{3n-1}{5n-2}$

(ii) The 10th term: $\frac{3(10)-1}{5(10)-2} = \frac{29}{48}$

Example:

A school hall has many chairs that often need to be stacked away.

Each chair that is added to this stack makes it 8cm taller. One chair is 55cm tall.

(i) Use your knowledge of patterns to find how high a stack of chairs will be that has 8 chairs in it.

(ii) How many chairs would there be if the stack was 127cm high?

Use the following table to help you.

Number of chairs	1	2	3	4	5
Height (cm)					

Teaching notes:
Point out that we need the general term to answer these questions.
Ask: *What is the first term in the sequence?*
(55)
Ask: *What do we need to add to this term to find the following term?*
(8)
Say: *Once the general term has been found, it can be used to answer both questions.*

Solution:

Number of chairs	1	2	3	4	5
Height (cm)	55	63	71	79	87

General term:
55 ; 63 ; 71 ; 79…
Common difference: 8

$$T_n = bn + c$$
$$\therefore T_n = 8n + c$$

Use term 1 to find c:

$$T_1 = 8(1) + c$$
$$55 = 8(1) + c$$
$$47 = c$$
$$\therefore T_n = 8n + 47$$
$$T_n = 8n + 47$$
$$T_8 = 8(8) + 47$$
$$T_8 = 111$$

8 chairs will be 111cm high

$$T_n = 8n + 47$$
$$127 = 8n + 47$$
$$80 = 8n$$
$$10 = n$$

There are 10 chairs in a stack that is 127cm high

This would be the ideal time to allow learners to do the exercises in their own textbooks.
What follows is work involving other types of patterns that learners should be made aware of and that will assist them in understanding patterns better.
If you have learners who find mathematics a challenge, it may be more beneficial to give them more exercises on linear patterns.

29. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

30. Give learners an exercise to complete on their own.

31. Walk around the classroom as learners do the exercise. Support learners where necessary.

<u>Describing and extending other patterns</u>

32. Tell learners that although linear patterns are the key focus in Grade 10, there are other patterns that they need to know about.

33. Say: *Write each of the following patterns in your book. Leave a few lines between each pattern.*

2 ;4 ;8 ;16….

1 ;4 ;9 ;16….

1 ;8 ;27 ;64…

1 ;1 ;2 ;3 ;5 ;8…

34. Say: *Study the patterns and add the next three terms.*

35. After a few minutes, ask:
What terms did you add? Describe how you found what the next terms would be.

Solutions:

2 ; 4 ; 8 ; 16 ; 32 ; 64 ; 128	Each term needed to multiplied by 2 to get the following term. This is known as a geometric sequence – there is a common ratio (in this case the common ratio is 2).
1 ; 4 ; 9 ; 16 ; 25 ; 36 ; 49	These are the perfect square numbers. Learners should recognise them.
1 ; 8 ; 27 ; 64 ; 125 ; 216 ; 343	These are the perfect cube numbers. Learners should recognise them.
1 ; 1 ; 2 ; 3 ; 5 ; 8 ; 13 ; 21 ; 34	This is the Fibonacci sequence. After the first 2 terms, the previous two terms are added to get the following term. (1 + 1 = 2 ; 1 + 2 = 3 ; 2 + 3 = 5 …)

36. Ask learners if they have any questions.

37. If time permits, allow learners to develop their own patterns and ask a fellow learner to try and describe the pattern and extend it by a few more terms.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=UuceRRQGk8E

(Linear sequences – nth term)

https://www.youtube.com/watch?v=KxHNQpRNyuA

(Triangular numbers)

https://www.youtube.com/watch?v=TPwOKOwWTQg

(Pentagonal numbers)

TERM 1, TOPIC 3, LESSON 2

REVISION AND CONSOLIDATION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners will have revised: • number patterns.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the first question on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	59 59	Rev	52	W/sh	59	3.4 3.5 3.6	65 69 70	3.2	68

CONCEPTUAL DEVELOPMENT

C

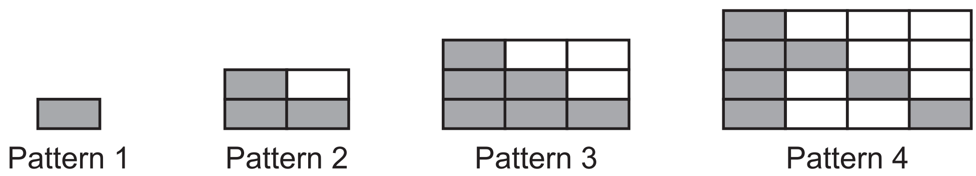
INTRODUCTION

1. This is a short topic, so learners shouldn't need too much time to recap all the concepts taught.
2. Doing examples from past examination papers will allow learners to consolidate their understanding as well as be exposed to how questions could be asked in an examination.

DIRECT INSTRUCTION

1. Tell learners that before they do a revision exercise on number patterns, you are going to work through a past examination paper question with them.
2. Learners should write the question in their books and make notes as they do so.

Dark tiles (D) and light tiles (L) are used to create patterns on a floor.
The first four patterns are shown below. For the patterns that follow, the tiles are arranged in a similar manner.



a) How many dark tiles were used in pattern 5?
b) How many light tiles were used in pattern 6?
c) Write down the general term (D_n) for the number of dark floor tiles used in each pattern.
d) Write down the general term (L_n) for the number of light floor tiles used in each pattern.
e) Which pattern will have exactly 64 light floor tiles?

NOV 2016

Teaching notes:

Learners should be able to count to answer the first two questions. Learners may want to sketch the pattern to assist them.

It would also be acceptable for learners to find the general term and answer the questions in that way.

c)

In order to answer this question, learners need to realise that the pattern formed by the number of dark tiles is a linear sequence.

Once learners realise this, it should be straightforward for them to find the common difference and hence the general term.

d)

Say: *Write down the number of light tiles as a sequence.*

(0 ; 1 ; 4 ; 9…)

Say: *You should recognise that 1, 4 and 9 are perfect squares.*

The general term for a perfect square is n^2. (Each natural number is squared to find the set of perfect squares).

The fact that zero is included on this list causes a small difficulty.

If we want to ensure that zero is the first term of the sequence, we will need to 'go back' one term (subtract 1) for our general term.

This will give $(n - 1)^2$

e)

Ask: *What is given and what is required?*

(Given the term in the sequence and looking for the position.)

Solutions:

a) 9 dark tiles

b) 25 light tiles

c) 1 ;3 ;5…

Common difference: 2

$$\therefore D_n = 2n + c$$

Use term 1 to find c:

$$D_1 = 2(1) + c$$
$$1 = 2(1) + c$$
$$-1 = c$$

$\therefore D_n = 2n - 1$

d) $L_n = (n - 1)^2$

e) $L_n = (n - 1)^2$

$64 = (n - 1)^2$

$64 = n^2 - 2n + 1$

$0 = n^2 - 2n - 63$

$0 = (n - 9)(n + 7)$

$n = 9$ or $n = -7$

$\therefore n = 9$

3. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

4. Give learners an exercise to complete on their own.

5. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 1, Topic 4: Topic Overview

EQUATIONS AND INEQUALITIES

A

A. TOPIC OVERVIEW

- This topic is the fourth of five topics in Term 1.
- This topic runs for two weeks (9 hours).
- It is presented over seven lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- The topic Equations (and Algebra) counts 30% of the final Paper 1 examination.
- The purpose of learning to solve Equations is also to help with Problem Solving which is the basis of all mathematics.

Breakdown of topic into 7 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Linear equations	1	5	Linear inequalities	1
2	Quadratic equations	1.5	6	Word problems	1
3	Simultaneous equations	1.5	7	Revision and Consolidation	2
4	Literal equations	1			

SEQUENTIAL TABLE

B

Senior phase	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Solve equations by using additive and multiplicative inverses and the laws of exponents • Determine the numerical value of an expression by substitution. • Use substitution in equations to generate tables of ordered pairs • Solve equations to include the use of factorisation and equations of the form where a product of factors = 0	Solve: • Linear equations • Quadratic equations • Literal equations • Linear inequalities • System of linear equations • Word problems • (Exponential equations – covered in the previous topic)	Solve: • Quadratic equations • Quadratic inequalities • Equations in two unknowns where one is linear, and one is quadratic • Exponential equations • Logarithmic equations

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Equations and Inequalities.

These include:

- a lack of understanding of inequalities
- factorising skills.

It is important that you keep these issues in mind when teaching this section.

While teaching Equations and Inequalities, use more than one method (particularly for inequalities) so learners can choose the method they understand best. Encourage learners to check their answers to equations.

D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test, with memorandum, and an investigation, with a rubric are provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of solving equations algebraically.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
linear equation	An algebraic equation in which each term has an exponent of one and the graphing of the equation results in a straight line. When solved, there is only one possible solution
quadratic equation	An equation of the second degree, meaning it contains at least one term that is squared. The standard form is $ax^2 + bx + c = 0$ a, b, and c are constants, or numerical coefficients, and x is an unknown variable. When solved, there are two possible solutions
literal equation	An equation with two or more variables
inequality	A mathematical sentence that uses symbols such as $<$, $\leq$, $>$ or $\geq$ to compare two quantities
solution	The value of the variable that makes an equation true
roots	A real number x will be called a solution or a root if it satisfies the equation. The roots are the x-intercepts of a function, if the equation is drawn on a Cartesian plane

formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value For example: Area = $l \times b$. This formula finds area of a rectangle and only measurements can replace the l *and* b
like terms	Terms that have the same variables. For example: $2a$ and $4a$ are like terms and can be added or subtracted
inverse operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other Multiplication and Division are the inverse operation of each other
identity	An equation that is true for any values that replace the variable. That means the variable can be any real number

TERM 1, TOPIC 4, LESSON 1

LINEAR EQUATIONS

Suggested lesson duration: 1 hour

A

POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners should be able to: • solve linear equations and know how to check their answer.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the four examples on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 2	61 63	1 2	57 58	7.1	61	4.1 4.2 4.3	73 74 76	4.1	78

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have been solving equations algebraically throughout the Senior Phase. This lesson should be revision for them.
2. Ensure all learners are confident with solving equations algebraically.

DIRECT INSTRUCTION

1. Ask learners to solve the following equations.

These are equations they have dealt with in Grade 8 and 9 and should be able to solve. Walk around the classroom while learners are working through them and assist where necessary. Take note of the setting out and correct it when necessary. Discuss any incorrect setting out when you do all the examples in full on the chalkboard once learners have had the chance to complete the questions on their own.

2. Examples:

Solve for the unknown:	Teaching notes	Solutions:
Remind learners that to solve any linear equation, the aim is always to get the variable on its own. Inverse operations are used to do this. Remind learners that whatever is done on one side of an equation must always be done on the other side to ensure the equation remains balanced.		
$-2x + 4 = 10$	Ask learners to tell you what needs to be 'removed' first and how that will be done. (To remove +4, subtract 4; to remove the × –2, ÷ by –2)	$-2x + 4 - 4 = 10 - 4$ $-2x = 6$ $\frac{-2x}{-2} = \frac{6}{-2}$ $x = -3$
$4 - 4x = 8 + 2x$	Ask learners to tell you what needs to be 'removed' first and how that will be done. Subtract $-2x$ from both sides and subtract 4 from both sides. Collect like terms. Divide both sides by –6	$4 - 4x = 8 + 2x$ $-4x - 2x = 8 - 4$ $-6x = 4$ $\frac{-6x}{-6} = \frac{4}{-6}$ $x = \frac{-2}{3}$

$5(x-1)=2(x+3)$	First use the distributive law to remove the brackets. Subtract $2x$ from both sides and add 5 to both sides. Collect like terms. Divide both sides by 3	$5(x-1)=2(x+3)$ $5x-5=2x+6$ $5x-2x=6+5$ $3x=11$ $x=\frac{11}{3}$
$a-4=\frac{2}{3}a+4$	Tell learners that it is good practice to remove fractions at the beginning of an equation that has one or more fractions in it. To do this, the lowest common denominator is required. Each term can be multiplied by the LCD (to keep the equation balanced).	$a-4=\frac{2}{3}a+4$ LCD = 3 $3a-12=2a+12$ $3a-2a=12+12$ $a=24$

3. Tell learners that it is always a good idea to check their answers when solving equations. This is one section in the curriculum where it is possible to check their answers and be sure that they are correct. If they are incorrect they can correct their work.

4. Check one of the above examples with learners now to demonstrate.

   Example 2 from above: $4-4x=8+2x$ Check $x=\frac{-2}{3}$

LHS:	$4-4x$ $=4-4\left(\frac{-2}{3}\right)$ $=4+\frac{8}{3}$ $=\frac{12}{3}+\frac{8}{3}$ $=\frac{20}{3}$	RHS:	$8+2x$ $=8+2\left(\frac{-2}{3}\right)$ $=8-\frac{4}{3}$ $=\frac{24}{3}-\frac{4}{3}$ $=\frac{20}{3}$
LHS = RHS $\therefore x=\frac{-2}{3}$ is correct			

5. Ask learners if they have any questions before spending more time on equations with fractions.

6. Discuss WHY it is acceptable to 'get rid' of fractions in equations but it is not acceptable in expressions. Answer: The equal sign allows us to change the look of the equation because we can perform any operation providing we do it on both sides to keep it balanced and therefore true.

Do the following two examples of solving equations involving fractions in full with learners.

<table>
<tr>
<td>Solve for x:
$\frac{x}{4} - \frac{2x}{3} = 5$
LCD = 12
$\frac{12}{1} \times \frac{x}{4} - \frac{12}{1} \times \frac{2x}{3} = 5 \times 12$
$3x - 8x = 60$
$-5x = 60$
$x = -12$</td>
<td rowspan="2">Discuss each step as you do the solution in full. Remind learners to write the steps down as well as the solution for their own reference later.
Steps:
• Find the Lowest Common Multiple of the denominators (commonly known as the Lowest Common Denominator -LCD).
• Multiply EACH term (across the entire equation) by the LCD.
• There should be no fractions at all after the previous step.
• Solve as usual.</td>
</tr>
<tr>
<td>$\frac{x+3}{2} + \frac{x-2}{3} = x - 2$
LCD = 6
$\frac{6}{1} \times \frac{x+3}{2} + \frac{6}{1} \times \frac{x2}{3} = 6 \times x - 6 \times 2$
$3(x + 3) + 2(x - 2) = 6x - 12$
$3x + 9 + 2x - 4 = 6x - 12$
$5x + 5 = 6x - 12$
$5x - 6x = -12 - 5$
$-x = -17$
$x = 17$</td>
</tr>
</table>

7. Once the examples are complete, ask learners if they have any questions.

8. Remind learners what you discussed about only being able to remove fractions from equations. Write the following algebraic fraction question on the board to illustrate when removing fractions is not possible.
   Learners should write the examples in their books and make notes as they do so.

<table>
<tr>
<td>Simplify: $\frac{x+3}{2} + \frac{x-2}{3} + x - 2$
$= \frac{3(x+3) + 2(x-2) + 6x - 12}{6}$
$= \frac{3x + 9 + 22x - 4 + 6x - 12}{6}$
$= \frac{11x - 7}{6}$</td>
<td>Point out how similar this looks to the last example. Point out that it is not an equation and we are not solving for x.
We cannot find the LCD and ‘remove’ fractions.
When adding and subtracting fractions, we need to find the lowest common denominator to be able to add and subtract. As they stand now, the fractions are unlike terms.
Note that the denominator remains throughout all the steps as well as in the answer.</td>
</tr>
</table>

9. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

10. Give learners an exercise to complete on their own. Remind learners to spend some time checking their answers. Doing this adds the advantage of not only knowing if they have made an error, but allows them to practice their algebraic skills.

11. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wShnYemIr28

https://www.youtube.com/watch?v=TkL0Iqs9mpY

https://www.youtube.com/watch?v=JDVRQWA5NZk

https://www.youtube.com/watch?v=KwLWAyDVAV4

TERM 1, TOPIC 4, LESSON 2

QUADRATIC EQUATIONS

Suggested lesson duration: 1,5 hours

A

POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners will have revised: • solve quadratic equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the first two examples from point 2 on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5	72	4 5 6	62 63 64	7.2	62	4.5	81	4.2	80

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners were introduced to quadratic equations in Grade 9.
2. From Grade 11, quadratic equations will be the focus of any solving of equations. Ensure learners get enough practice to give them confidence in this topic.

DIRECT INSTRUCTION

1. Ask: What is a quadratic equation? Learners should write the definition in their books. [An equation of the second degree, meaning it contains at least one term that is squared. The standard form is $ax^2 + bx + c = 0$ where a, b, and c are constants, or numerical coefficients, and x is an unknown variable. When solved, there are two possible solutions].
2. Discuss:

$$x(x + 2) = 0$$

Note the product (the bracket indicates multiplication) of two values (x and (x+2)) is equal to zero. The only way this is possible is if one of those values is zero.

$$\therefore\ x = 0 \qquad \text{OR} \qquad x + 2 = 0$$

The second part does not yet show a final solution so it needs to be completed using knowledge of solving equations.

$$\therefore\ x = 0 \qquad \text{OR} \qquad x = -2$$

Note that either value can be substituted into the equation to test if either one of them will work:

$$x(x + 2) = 0$$

If $x = 0$:
LHS $= 0(0 + 2)$
$= 0(2)$
$= 0 =$ RHS

If $x = -2$:
LHS $= -2(-2 + 2)$
$= -2(0)$
$= 0 =$ RHS

This shows that both solutions will work.

$(x + 3)(x - 4) = 0$

$\therefore\ x + 3 = 0 \quad \text{OR} \quad x - 4 = 0$

$\therefore\ x = -3 \quad \text{OR} \quad x = 4$

Point out that in both examples, the expressions on the left-hand side were already factorised and that this is the key to solving quadratic equations.
Learners often multiply out when a question is given in its factorised form and sadly, this is not always done correctly. Tell learners that if the left-hand side is already factorised and equal to zero, there is no need to multiply out, then try and factorise again.

3. Move onto equations in which it is necessary for us to factorise before being able to solve. Point out that their factorising skills will be required and this will be an opportunity to practice them again.

4. Discuss this basic example:

Solve $x^2 = 25$
The exponent ('squared') shows that there are TWO solutions that would make this statement true.
When working with the theorem of Pythagoras, learners were taught to find the square root of both sides - which gave them an answer of 5. The reason this was allowed is that the theorem deals with lengths of sides and it is impossible for a length or a distance to be anything other than positive.
However, there is in fact one other number that can be squared to give 25 and that is '–5'
This equation will be solved in full later.

5. Learners should write the following steps in their books. They can then refer to them as you do examples together and while completing an exercise on their own.

To solve quadratic equations, follow these steps:
- Ensure all terms are on one side (usually the left-hand side) of the equal sign and zero on the other.
- Factorise the expression on the LHS (left hand side).
- The factorised expression should be made up of two factors.
- If two factors multiply to make zero, then either one of these factors needs to equal zero (any number multiplied by zero always equals zero).
- State the two options, making each factor equal to zero and solve each equation.

6. Learners should follow the steps as you do some examples with them which they should write in their books.
If a step is not clear, encourage learners to ask for further explanation.

Solve for x:	Teaching notes	Solutions:
$x^2 = 25$	Ask: what do we need to do first? (subtract 25 from both sides to get zero on the right-hand side) Say: now we need to factorise. Ask: how can we factorise? (difference of two squares)	$x^2 = 25$ $x^2 - 25 = 0$ $(x + 5)(x - 5) = 0$ $\therefore\ x + 5 = 0$ or $x - 5 = 0$ $\therefore\ x = -5$ or $x = 5$
$x^2 - 2x = 0$	Ask: what do we need to do first? (factorise) Ask: how can we factorise? (take out a common factor)	$x^2 - 2x = 0$ $x(x - 2) = 0$ $\therefore$ x = 0 or $x - 2 = 0$ $x = 2$
$x^2 + 5x = 6$	Ask: what do we need to do first? (subtract 6 from both sides) Ask: how can we factorise? (trinomial)	$x^2 + 5x = 6$ $x^2 + 5x - 6 = 0$ $(x + 6)(x - 1) = 0$ $\therefore\ x + 6 = 0$ or $x - 1 = 0$ $x = -6$ or $x = 1$
$3x^2 + x - 10 = 0$	Ask: what do we need to do first? (factorise) Ask: how can we factorise? (trinomial)	$3x^2 + x - 10 = 0$ $(3x - 5)(x + 2) = 0$ $\therefore 3x - 5 = 0$ or $x + 2 = 0$ $x = \frac{5}{3}$ or $x = -2$

7. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

8. Give learners an exercise to complete on their own. Remind learners to consult their summary of what to do while working on their own.

9. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=g6RnAY_VkMs

https://www.youtube.com/watch?v=SDe-1IGeS0U

TERM 1, TOPIC 4, LESSON 3

SIMULTANEOUS EQUATIONS

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners should be able to: ● solve simultaneous equations.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. Write the equations from points 1 and 2.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	69	8	68	7.5	67	4.6 4.7 4.8 4.9	83 85 87 88	4.3	88

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Simultaneous equations can be used to solve everyday problems, especially those that are more difficult to think through without writing anything down.
2. Simultaneous equations are a new concept for most learners.

DIRECT INSTRUCTION

1. Start the lesson by writing $x + y = 9$ on the chalk board.
   Ask: *What could x and y be?*
   Learners may say 3 and 6 or 5 and 4. Allow learners to give a few answers. Tell them they could all be correct.
2. Write $2y - x = 15$ on the chalkboard alongside $x + y = 9$. Tell learners that we need to solve for x and y so that BOTH equations will be true.
3. Ask: *Can you see that now we need to approach this a little differently?*
   There are two methods to solve simultaneous equations. Learners should write the methods in their books and make notes as they do so.

Solve for x and y: $2y - x = 15$ and $x + y = 9$
When doing this example, discuss with learners as you proceed. Once the example is completed, learners can write down the steps for future reference.
Substitution method – teaching notes Tell learners to choose any equation to start with where it is easy to get one of the variables on its own. Where possible, this should not include a need to work with fractions which will make the calculations more complex than they need be. In the case of this question there are three options. The x in equation 1 (the y has a coefficient of 2 so it would make a fraction) and the x or y in equation 2. Say: *We will use equation 2 and solve for x*
$x + y = 9$ $x = 9 - y$
Once this has been done, say: *Now we need to use this information in the OTHER equation. $9 - y$ will be substituted for x. This will create an equation with only one unknown (y) and is possible to solve.*

$$2y - x = 15$$
$$2y - (9 - y) = 15$$
$$2y - 9 + y = 15$$
$$3y - 9 = 15$$
$$3y = 24$$
$$y = 8$$

Say: *We now have a value for one of the variables. This can be used in the equation previously used to find the second variable.*

$$x = 9 - y$$
$$x = 9 - 8$$
$$x = 1$$

Tell learners that using this method is a little like a see-saw. First use one equation, then the other, then back to the first equation.

Encourage learners to check their answers.

$2y - x = 15$	$x + y = 9$

If $x = 1$ and $y = 8$, both equations should work with these values.

LHS = 2(8) – 1	LHS = 1 + 8
= 16 – 1 = 15 = *RHS*	= 9 = *RHS*

Elimination method – teaching notes

Tell learners that this method works particularly well if the coefficients of the same variables are the same or the additive inverse of each other.

For example, if one equation had a $3y$ then this method would work well if the 2nd equation also had a $3y$ or a $-3y$.

If the coefficients are not as mentioned above, it is still possible to use this method, but we need to multiply each term of at least one equation by a factor that will make the above scenario true. If this is the case, the substitution method is usually less work.

Say: *In this example, the x has a coefficient of* 1 *in one of the equations and a coefficient of* –1 *in the other equation.*

To use this method, we need to place the equations underneath each other with like terms below one another. We might need to rearrange the terms in one of the equations.

Remind learners that the sign to the left of a term belongs to it and must move with that term.

$2y - x = 15$ and $x + y = 9$

$$2y - x = 15$$
$$y + x = 9$$

Say: *Once this has been completed, we either add or subtract the equations in the columns of like terms. We choose the operation that will eliminate one of the variables – in this case the x's.*

As the signs are different, adding will eliminate the x's.

$$\begin{aligned} 2y - x &= 15 \\ \underline{y + x} &\underline{= 9} \\ 3y &= 24 \\ y &= 8 \end{aligned}$$
Say: *Now that we have a solution, we can use any one of the original equations to find the 2nd solution.*
$$\begin{aligned} x + y &= 9 \\ x + 8 &= 9 \\ x &= 1 \end{aligned}$$

4. Tell learners to write the following summary of steps into their books:

Substitution method	Elimination method
• Get ONE of the variables by itself in ONE of the equations. • Use this information to substitute back into the second equation. You should now have an equation with only one unknown variable. • Solve for this variable. • Use the information just found to substitute back into the first equation and solve for the second variable.	• Write one equation below the other ensuring all the same terms are underneath each other. • If the coefficients have the same sign, subtract one equation from the other to eliminate that variable. OR • If the coefficients have opposite signs, add the two equations to eliminate one of the variables. • Solve the 'new' equation which has only one unknown. • Use the information just found to substitute back into one of the original equations to solve for the missing variable.

5. Ask if there are any questions before you do two more fully worked examples.

6. Tell learners to write the following example in their books.

<table>
<tr><td colspan="2">Solve the following equations simultaneously:
$x + 2y = 5$ and $x - y = -1$</td></tr>
<tr><td>Elimination:
$x + 2y = 5$
$\underline{-(x - y = -1)}$
$3y = 6$
$y = 2$
$x - y = -1$
$x - 2 = -1$
$x = 1$
$\therefore\ x = 1$ and $y = 2$</td><td>Say: Check the coefficients of both variables in both equations.
Ask: Which one is helpful?
(The x's both have a coefficient of 1).
Ask: Will we need to add or subtract the equations to eliminate the x's?
(Subtract).
Remind learners to be careful when subtracting as all the terms will be affected by the subtraction sign. It is a good idea to use brackets.
Once the first variable has been solved, ask: What needs to be done now?
(Use the information in any equation to solve for the 2nd variable).</td></tr>
<tr><td>Substitution:
$x + 2y = 5$
$x = -2y + 5$

$x - y = -1$
$-2y + 5 - y = -1$
$-3y = -1 - 5$
$-3y = -6$
$y = 2$

$x + 2y = 5$
$x + 2(2) = 5$
$x = 1$</td><td>Ask: Is it possible to start with any of the equations to get a variable on its own easily?
(Yes – the x in equation 1 and the x or y in equation 2).
Say: for this example, we will use the first equation.
Once this has been done, ask:
What do we need to do next?
(Use the other equation and substitute the new information in then solve for the other variable).
Once this has been done, ask:
What do we need to do next?
(Use the first equation again to solve for the second variable).</td></tr>
</table>

7. Do one more example. Before you do it, tell learners that they can try it on their own first, choosing the method they prefer. Once learners have had enough time to complete the question, do it in full on the board. Use both methods. Tell learners to correct their own method and copy down the method that they did not use.

Solve for x and y: $6x + y = 22$ and $4x - y = 8$	
Substitution method: $6x + y = 22$ and $4x - y = 8$ $6x + y = 22$ $y = 22 - 6x$ $4x - y = 8$ $4x - (22 - 6x) = 8$ $4x - 22 + 6x = 8$ $10x = 30$ $x = 3$ $6x + y = 22$ $6(3) + y = 22$ $18 + y = 22$ $y = 4$	Elimination method: $6x + y = 22$ and $4x - y = 8$ $6x + y = 22$ $\underline{+(4x - y = 8)}$ $10x = 30$ $x = 3$ $6x + y = 22$ $6(3) + y = 22$ $18 + y = 22$ $y = 4$

8. Ask learners if they have any questions.

9. Now that you have worked through the algebraic solving of simultaneous equations, discuss what the solutions of a set of two linear simultaneous equations represent. Remind learners of the straight-line graph that they learnt about in Grade 9. Use the previous examples (ask them to turn back to the first example in their own books), say: The two equations are in the form $y = mx + c$ and would form straight line graphs.
   Point out that if the two lines were drawn on a Cartesian plane the solutions found represent the coordinate for the point of intersection of the two lines.

If there is time, plot the two lines from one of the examples and show that the point of intersection is represented in the solutions. This would be ideal. A visual understanding of the solving of equations leads to a better understanding.

10. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=Lwto-lQzmec

https://www.youtube.com/watch?v=7sqDS-PvGEI

(Elimination method)

https://www.youtube.com/watch?v=8ockWpx2KKI

(Substitution method)

TERM 1, TOPIC 4, LESSON 4

LITERAL EQUATIONS

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners should be able to: • solve literal equations.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
6	74	10 11	73 73	7.4	66	4.4	78	4.5	90

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners who do Physical Science will have worked with literal equations already. Learners who do not do Physical Science may find this section more challenging.
2. Manipulating a formula is a useful skill to have for many mathematical topics. Ensure learners get enough practice before moving on. This topic will not be covered as a full lesson again in the FET phase.

DIRECT INSTRUCTION

1. Start the lesson by writing a formula well known to learners on the chalkboard:

$$V = lbh$$

2. Ask: *What is this formula used for?*
   (Finding the volume of a rectangular prism).
   Say: *This is the layout of the formula that we are used to seeing. In general, we are given the length, breadth and height of a prism and are asked to find the volume. The formula allows simple substitution to find the answer.*
   However, what if we were given the volume, length and height and asked to find the breadth. Can you see that would be more complicated?
   We could substitute and then work with the equation to get the 'b' on its own but we could also manipulate the formula before we substitute. This is what we will do today. Solving literal equations means to make one variable the subject of the formula – in other words, get one variable on its own while any other variables will be on the other side of the equal sign.

3. Say: *Let's looks at the formula for finding volume of a rectangular prism again and solve for b. Don't be put off by other variables, work as you always have to get a variable by itself.*
   Ask: *What is 'in the way' of the b being on its own?*
   (Times l and times h).
   How can we use inverse operations to we remove these?
   (Divide each side by lh)

$$V = lbh$$

$$\frac{V}{lh} = \frac{lbh}{lh}$$

$$\frac{V}{lh} = b$$

Say: *We now have a formula for finding the breadth of a rectangular prism given the volume, height and length.*

4. Say: *The equations given will not always be an accepted formula. It may just be an equation with several variables in it. However, the method for solving the equation is the same – we will use inverse operations to get a certain variable on its own.*
5. Work through more examples now.
   Learners should write the examples in their books and make notes as they do so.

Examples:	Teaching notes:
Solve for x: $ax + b = c$ $ax = c - b$ $\frac{ax}{a} = \frac{c-b}{a}$ $x = \frac{c-b}{a}$	Ask: *What is 'in the way' of getting the x on its own?* ($+b$ and $\times a$) Say: *We need to use inverse operations.*
Solve for p: $a(p - b) = b(p + c)$ $ap - ab = bp + bc$ $ap - bp = bc + ab$ $p(a - b) = bc + ab$ $\frac{p(a-b)}{a-b} = \frac{bc+ab}{a-b}$ $p = \frac{bc+ab}{a-b}$	Ask: *What needs to be done first?* (Use the distributive law to multiply and remove the brackets). Say: *Note that the variable we are solving for appears in TWO terms.* *When this is the case, we need to get the terms containing a variable on one side and all terms without the variable we are solving for on the other side.* *Once this has been done, you will need to factorise to get the variable on its own.* *Inverse operations can then be used.*
Make a the subject of the formula: $S = \frac{n}{2}(a + l)$ $2 \times S = 2 \times \frac{n}{2}(a + l)$ $2S = n(a + l)$ $\frac{2S}{n} = \frac{n(a+l)}{n}$ $\frac{2S}{n} = a + l$ $\frac{2S}{n} - l = a$	Ask: *What is 'in the way' of getting the a on its own?* ($\div 2$, $\times n$, $+l$) Say: *We need to use inverse operations.*

Make s the subject of the formula: $v^2 = u^2 + 2as$ $v^2 - u^2 = 2as$ $\frac{v^2 - u^2}{2a} = \frac{2as}{2a}$ $\frac{v^2 - u^2}{2a} = s$	Ask: *What is 'in the way' of getting the s on its own?* ($+u^2$, $\times 2a$) Say: *We need to use inverse operations.*
Make u the subject of the formula: $v^2 = u^2 + 2as$ $v^2 - 2as = u^2$ $\sqrt{v^2 - 2as} = \sqrt{u^2}$ $\sqrt{v^2 - 2as} = u$	Ask: *What is 'in the way' of getting the s on its own?* ($-2as$, the square) Say: *We need to use inverse operations.* Ask: *What is the inverse operation of squaring?* (Finding the square root).

5. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

6. Give learners an exercise to complete on their own.

7. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=L2e3LPhAXW8

https://www.youtube.com/watch?v=LbyrG4kGG6Y

https://www.youtube.com/watch?v=VRurdA9Pq6s

TERM 1, TOPIC 4, LESSON 5

LINEAR INEQUALITIES

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners should be able to: • solve linear inequalities • represent inequalities on a number line and in interval notation.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the table from point 1 with the headings ready. You will populate the table while discussing with learners.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	64	12 – 16	77 – 80	7.5	67	4.11 4.12	95 96	4.6	100

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. If learners are confident with solving equations, they should manage the solving of inequalities.
2. Spend time discussing inequalities and that the list represented could be extensive – this is the reason for the solid line used when representing the inequality on a number line, assuming that the variable is an element of real numbers.

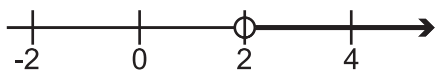
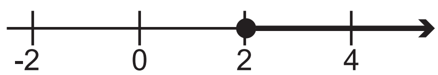
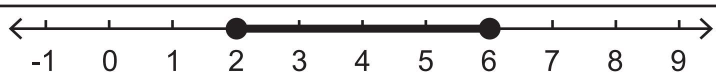
DIRECT INSTRUCTION

1. Revise representing real numbers on a number line from earlier in the term. (This is Resource 3 in the Resource Pack).

Inequality sign	words	Open/closed dot	Arrow points to the
$x >$	Greater than	Open $\circ\!\!\longrightarrow$	right
$x \geq$	Greater than or equal to	Closed $\bullet\!\!\longrightarrow$	right
$x <$	Less than	Open $\longleftarrow\!\!\circ$	left
$x \leq$	Less than or equal to	Closed $\longleftarrow\!\!\bullet$	left

As you are writing, explain that the open dot shows that a certain number is NOT included in the list whereas a closed dot is used to show that a certain number IS included in the list.

2. Point out that the direction of the inequality is the same as the direction of the arrow.
3. Use the following examples to explain this further. Do the examples one at a time and discuss each one as it is completed. Remind learners what the open and closed dots mean.

Inequality		Interval notation
$x > 2$		$x \in (2\ ;\ \infty)$
$x \geq 2$		$x \in [2\ ;\ \infty)$
$2 \leq x \leq 6$		$x \in [2\ ;\ 6]$

$2 < x < 6$		$x \in (2 ; 6)$
$2 \leq x < 6$		$x \in [2 ; 6)$
$2 < x \leq 6$		$x \in (2 ; 6]$

4. Tell learners that when we solve inequalities we need to be able to represent the solutions on a number line or in interval notation.

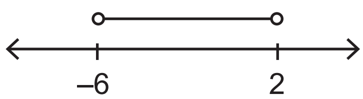
5. Discuss the main difference between solving equations and inequalities. The methods are the same – to use inverse operations to get the variable on its own. However, if there is a need to multiply or divide by a negative number, the inequality signs need to be reversed.

6. Explain why this is the case:
   Write $-10 < 2$ on the board
   Ask: *Is this statement true?*
   (Yes)
   Tell learners to multiply both sides by -1
   Ask: *Is the statement still true?*
   (No: $10 < -2$ is not true)
   Point out that if learners change the inequality sign, the statement will still be true.

$$10 > -2$$

   Write these examples on the board and do in full with the learners.
   Learners should write the examples in their books and make notes as they do so.

Example:	Teaching notes:
Solve the inequality: $3(x + 2) \leq 12$ $3x + 6 \leq 12$ $3x \leq 6$ $\frac{3x}{3} \leq \frac{6}{3}$ $x \leq 2$	Remind learners to solve an inequality the same way they would solve an equation. Use inverse operations and algebraic rules to get the x by itself.
Solve the inequality: $\frac{-2x}{5} + x > 2x - 3$ LCD = 5 $-2x + 5x > 10x - 30$ $-7x > -30$ $\frac{-7x}{-7} < \frac{-30}{-7}$ $x < \frac{30}{7}$	Ask: *What do we need to do first?* (Find the LCD and multiply throughout to remove fractions). Remind learners that they need to change the inequality sign when dividing by a negative number.

Solve the inequality: $-2 < \frac{2x}{3} + 2 \leq 1$ $-2 - 2 < \frac{2x}{3} + 2 - 2 \leq 1 - 2$ $-4 < \frac{2x}{3} \leq -1$ $-4 \times 3 < \frac{2x}{3} \times 3 \leq -1 \times 3$ $-12 < 2x \leq -3$ $\frac{-12}{2} < \frac{2x}{2} \leq \frac{-3}{2}$ $-6 < x \leq \frac{-3}{2}$	Point out that this inequality represents a starting and ending point. Remind learners that this type of inequality is made up of two and statements. For example, $x < 2$ and $x > -1$ This can be written as: $-1 < x < 2$ This represents all the numbers between –1 and 2. Say: *The strategy is similar to the previous examples, but to get x on its own in the centre will require treating the outer terms the same way.* Ask: *What is 'in the way' of getting the x on its own?* (+2, ÷3, ×2).
Solve the inequality and represent your answer on a number line. $-1 < 3 - 2x < 15$ $-1 - 3 < 3 - 2x - 3 < 15 - 3$ $-4 < -2x < 12$ $\frac{-4}{-2} > \frac{-2x}{-2} > \frac{12}{-2}$ $2 > x > -6$ $-6 < x < 2$ 	Ask: *What is 'in the way' of getting the x on its own?* (+3, × –2) Point out that this is no longer in the correct format. The lowest number should be at the start of the inequality so this inequality needs to be reorganised. Say: *It is like sliding it out (to the right) and flipping it over.* Say: *Now we must represent this on a number line.* Ask: *Do we use an open or closed dot for the –6?* (Open because of the < sign). Ask: *Do we use an open or closed dot for the 2?* (Open because of the < sign).

Solve the following inequality and represent your answer in interval notation. $-3 \leq x + 1 < 6$ $-3 - 1 \leq x + 1 - 1 < 6 - 1$ $-4 \leq x < 5$ $\therefore x \in [-4; 5)$	Ask: *What is 'in the way' of getting the x on its own?* (–1). Say: *Now we need to represent this in interval notation.* Ask: *Do we use a round or square bracket for the -4?* (Square – because of the ≤. The 'or equal to' means the –4 is included). Ask: *Do we use a round or square bracket for the 5?* (Round – because of the <. Less than means the 5 is not included).

7. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

8. Give learners an exercise to complete on their own.

9. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=0X-bMeIN53I

https://www.youtube.com/watch?v=1raifwKcl5A

https://www.youtube.com/watch?v=y0R54UeqClo

TERM 1, TOPIC 4, LESSON 6

WORD PROBLEMS

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners should be able to: ● work with a partner to read, understand and develop a strategy to solve word problems.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the seven tips on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
		3 7 9	60 66 70	7.7	74	4.10	92	4.4	93

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Many learners find word problems difficult.
2. This lesson attempts to give learners as much assistance as possible to help them overcome their resistance to solving problems.
3. Solving problems is an integral part of life and an important skill to have.

DIRECT INSTRUCTION

1. Tell learners how important it is to have the ability to solve a problem.
2. Tell learners that you know how difficult this can be and that you will give them as many tools as possible to assist them in becoming more confident in solving word problems. There are a few different types of problems which tend to be more common than others at this level. We will be look at these one at a time.

   Note: Throughout the lesson, remind learners that the tips and steps you give them are not hard and fast rules. If they can solve any problem using a different strategy that is a positive sign – they must not feel that they must do it a certain way.

3. Start with some general guidelines for solving word problems. Tell learners to write these in their books.

   1. Read each problem three times. The first reading is to determine what the problem is, the second to identify the "maths" words and the third is to set up an equation.
   2. Decide exactly what is being asked and make this the variable ready to be represented in the equation. If there are two unknowns, make the smaller one the variable chosen.
   3. If two or more items are involved, express one item in terms of the other.
      For example: The boy is twice as old as his sister. The sister is younger, so she can be represented by the variable chosen (x being the most common) and the boy's age will then be $2x$.
   4. Set up your equation using any other information given in the statement.
   5. Solve the equation.
   6. Answer the question asked.
   7. Make sure the answer makes sense. For example, age cannot be negative.

4. Tell learners that we are going to look at five different types of questions and do an example for each one.
   Learners should write the examples in their books and make notes as they do so.

NUMBER QUESTIONS

5. Discuss some general points regarding numbers.
   Ask: *If a number is x, what will the next consecutive number be?*
   (x + 1)
   Ask: *If an even number is x, what will the next even number be?*
   (x + 2)
   Ask: *If an odd number is x, what will the next odd number be?*
   ((x + 2) – odd numbers are also 2 digits apart)

6. Do the following example on the board:

The sum of two consecutive numbers is 83. Find the numbers.	
$\therefore\ (x) + (x + 1) = 83$ Solve the equation: $x + x + 1 = 83$ $2x + 1 = 83$ $2x = 82$ $x = 41$ Answer the question: The two numbers are 41 and 42	Ask: *What is being asked?* (What the two numbers are). Let the first number be x. Ask: *What will the second number be, considering that the first number is x?* (x + 1). These two numbers must add together to make 83 Make the equation using this information and solve it.

AGE QUESTIONS

7. Share the following information with learners:
   A table is useful in these types of questions. There will always be some information about now and other information about some time in the past or the future. Now and the other time mentioned will be the columns in the heading and the two people involved will be the headings for the rows.
   Let the youngest person's age be the variable.

8. Do this example on the board and demonstrate this strategy:

A father is now 3 times as old as his son. Eight years ago, their combined age was 64 years. How old is the father now?

Ask: *What is being asked?*
(The father's age)
Say: *We could let the father's age be the unknown but then we would have to work with fractions. We can avoid this by letting the younger person's age be the unknown and using what we know to find the father's age at the end.*
Let the son's age be x.
Ask: *How can we represent the father's age, considering that the son's age is x?*
($3x$)
Ask: *How old was the son 8 years ago, considering that he is x years old now?*
($x - 8$)
Ask: *How old was the father 8 years ago, considering that he is 3x years old now?*
($3x - 8$)
Say: *Now that the table has been populated, we need to make an equation.*
To do this, we need to consider the information that represents the 'other time' (8 years ago) as it is during this time that the comparative information is given – 8 years ago their combined age was 64.

Person	**Now**	**8 years ago** (subtraction will be used)
Father	$3x$	$3x - 8$
Son	x	$x - 8$

$x - 8 + 3x - 8 = 64$
$4x - 16 = 64$
$4x = 80$
$x = 20$ That means the son is 20 and the father is 60 years old.
Check your answer by returning to the question and seeing whether the numbers 'work'.

MEASUREMENT

9. It is often useful to make a sketch in this type of question.

10. Do the following example on the board with learners.
    Learners should write the example in their books and make notes as they do so.

The length of a rectangle is 5cm more than the width. The perimeter is 160cm. Find the length.	
$P = 2l + 2w$ $P = 2x + 2(x – 5)$ $160 = 2x + 2x – 10$ $160 = 4x – 10$ $170 = 4x$ $170 = 4x$ $\frac{170}{4} = \frac{4x}{4}$ $42,5 = x$ The length is 42,5cm	Ask: *What is being asked?* (The length). Let the length be x Ask: *What will the width be considering the length is x?* (x – 5). Ask: *Perimeter is mentioned – how is the perimeter of a rectangle found?* (It is found by adding 2 lengths and 2 widths).

MONEY

11. Point out that when dealing with money, learners must be sure that they deal with rands only or cents only. If these units are used together, it is more difficult to get the correct answer.

12. Do the example on the board with learners.
    Learners should write the example in their books and make notes as they do so.

An amount of R34,80 is made up of 50c and 20c coins.
How many 50c coins are there out of a total of 120 coins?

A table to summarise all the information is useful in this situation.
Ask: *What is being asked?*
(Number of 50c coins).
Let the number of 50c coins be x.
Ask: *How many 20c coins are there, considering that the number of 50c coins is x?*
($120 - x$).
Note: if learners struggle with this idea, ask a few questions using actual values. For example, *if you have 10* 50c coins, how many 50c coins are there?
After discussing a few of these examples, learners should realise that each time they start with 120 and subtract the amount given.
Say: *Since coins and cents are used in the main part of the question, it is important that the total money be changed into cents too.*
Ask: *If we know that we have x 50c coins and $120 - x$ 20c coins, what do we know about the value of all these coins?*
(they add up to R34,80).
Say: *This will be the basis of the equation*

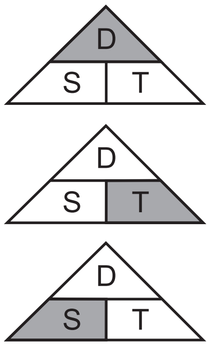
Value of coins	No of coins	Total value
50c	x	$50x$
20c	$120 - x$	$20(120 - x)$

$$50x + 20(120 - x) = 3480$$
$$50x + 2400 - 20x = 3480$$
$$30x + 2400 = 3480$$
$$30x = 1080$$
$$\frac{30x}{30} = \frac{1080}{30}$$
$$x = 36$$

There are 36 coins that are 50c coins.

SPEED, DISTANCE AND TIME

13. Learners need to know the relationship between distance, speed and time.
    Some learners like to use the triangle to help them:



Distance = Speed x Time

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

Show learners that if they cover the aspect they are looking for (here it is shaded), the required formula will be clear.

14. Secondly, tell learners that a table is a good tool for these questions as well.

15. Do this example on the board with learners.
    Learners should write the example in their books and make notes as they do so.

Two marathon runners set off at 6h00 in opposite directions. One runs at an average speed of 12km/h and the other runs at an average speed of 8km/h. At what time will they be 90km apart?

Tell learners to first draw up a table with two rows to represent the two runners and three columns for speed, distance and time.
Ask: *What is being asked?*
(Time – hours).
Let the time (number of hours) be x
Ask: *Will both runners' times be the same?*
(Yes – they will be 90km apart at the same time even though one runner will have run further).
Tell learners to fill in the time (x) on the table for each runner.
Ask: *Do we know anything else for sure that we can put on the table?*
(Yes – the speed of each of the runners).
Tell learners to fill the speed in on the table for each runner.

This is an important part of the process. Point out that we don't know the distance – only that the distance of each runner combined will add to 90km.

Point out that we do know a formula for distance once we have a value for speed and time.

Ask: *How do we find distance?*

(distance = speed × time).

Tell learners to multiply each runner's speed by his time and fill it in under distance.

Once the table has been populated, ask:

What information has not yet been used?

(The 90km).

Say: *This will be used to form an equation and solve for x.*

	speed	**distance**	**time**
Runner 1	12km/h	$12 \times x$	x
Runner 2	8km/h	$8 \times x$	x

$$12x + 8x = 90$$
$$20x = 90$$
$$\frac{20x}{20} = \frac{90}{20}$$
$$x = 4{,}5$$

It will take 4,5 hours for them to be 90km apart

∴ it will be 10h30

16. Remind learners that the types of questions and examples you have done with them by no means covers anything they could ever be asked. It is merely a guide to assist them. Learners should practice as many as possible. The more examples they try, the more confident they will become.

17. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

18. Give learners an exercise to complete with a partner.

19. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://learn.mindset.co.za/resources/mathematics/grade-10/equations-and-inequalities/word-problems/02-simultaneous-equations-word-problems

http://learn.mindset.co.za/resources/mathematics/grade-10/equations-and-inequalities/word-problems/01-dimension-and-speed-word-problems

http://learn.mindset.co.za/resources/mathematics/grade-10/equations-and-inequalities/word-problems/03-digit-problem

TERM 1, TOPIC 4, LESSON 7

REVISION AND CONSOLIDATION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	22
Lesson Objectives By the end of the lesson, learners will have revised: ● all aspects of solving equations and inequalities.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the first few questions on the board.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev S Ch	74 75	Rev	81	w/sh	80	4.13 4.14	99 100	4.7	101

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Point out issues that you know are important as well as problems that you encountered with your learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

The approach in this lesson is to do fully worked examples from a past paper. As you work through these examples with the learners, it is important to frequently talk about as many concepts as possible.

For example, use words and phrases such as *'keeping the equation balanced'* and *'what is done to one side must be done to the other side'* wherever possible, constantly reminding learners what they have already learnt.

1. Work through the fully worked examples with learners.
   Learners should write the examples in their books and make notes as they do so.

1. Solve for x:
   a) $x(x - 1) = 20$
   b) $\frac{3x - 2}{2} = x + 1$
2. Given: $-4 \leq -\frac{1}{2}m < 5$ where $m \in R$
   a) Solve for m.
   b) Write the answer to 2b) in interval notation.
3. Given $4x^2 - y^2 = 171$ and $2x - y = 9$
   a) Calculate the value of $2x + y$
   b) Solve simultaneously for x and y.

Solutions:	Teaching notes:
1a) $x(x - 1) = 20$ $x^2 - x = 20$ $x^2 - x - 20 = 0$ $(x + 4)(x - 5) = 0$ $x = 4$ or $x = -5$	Ask: *What must be done first?* (Multiply). Ask: *What do you notice once the multiplication has been done?* (It is a quadratic equation). *What must be done to solve this type of equation?* (Get all terms to one side and zero on the other. Factorise and find the 2 solutions).
1b) $\frac{3x - 2}{2} = x + 1$ $3x - 2 = 2(x + 1)$ $3x - 2 = 2x + 2$ $3x - 2x = 2 + 2$ $x = 4$	Ask: *What should we do when solving equations with fractions?* (Find the LCD and multiply all terms by it to remove the fraction).
2a) $-4 \leq -\frac{1}{2}m < 5$ $-8 \leq -m < 10$ $\frac{-8}{-1} \leq \frac{-m}{-1} < \frac{10}{-1}$ $8 \geq m > -10$ $-10 < m \leq 8$	Remind learners that a linear inequality is treated the same as an equation. UNLESS we need to divide by a negative integer, then the inequality sign will change. Say: *Remember, we need to get the m by itself in the centre. Whatever you do to one term to achieve this must be done to all terms.* Once you have got to this step: $8 \geq m > -10$ stop to discuss. Say: *Note that this is not the accepted layout of an inequality that has a starting and ending point. We need to reorganise it to ensure the smallest number is at the front. It is like sliding it out (to the right) and flipping it over.*
2b) $m \in (-10; 8]$	Ask: *What is interval notation?* (A method to represent all real numbers from an inequality). Ask: *What do we need to remember when using interval notation?* (Round brackets and square brackets – round means 'excludes' and square means 'includes').

3a) $4x^2 - y^2 = 171$ $(2x - y)(2x + y) = 171$ but $2x - y = 9$ $\therefore 9(2x + y) = 171$ $2x + y = 19$	This is not a straightforward solving question. Learners need to recognise the difference of two squares and factorise. Once this has been done, learners need to notice the second piece of information given and use substitution to simplify.
3b) $2x + y = 19$ and $2x - y = 9$ $2x + y = 19$ $y = 19 - 2x$ $2x - y = 9$ $2x - (19 - 2x) = 9$ $2x - 19 + 2x = 9$ $4x = 28$ $x = 7$ $2x + y = 19$ $2(7) + y = 19$ $14 + y = 19$ $y = 5$ Method 2 $2x + y = 19$ $\underline{-(2x - y = 9)}$ $2y = 10$ $y = 5$ $2x + y = 19$ $2x + 5 = 19$ $2x = 14$ $x = 7$	Say: *Remind us how to solve simultaneous equations.* (Get one variable on its own in one equation, substitute the new information into the second equation and solve for the unknown. Use this result to substitute back into the first equation and solve for the second variable). Or (Subtract the equations to eliminate the x's then solve for y. Use this solution to find x). Note: Do both methods with learners. Remind learners of the link to Functions. Say: *When you are solving simultaneous equations, you are actually finding the point(s) of intersection of two functions. These functions are both linear (straight lines) and therefore you will only find one set of solutions.*

2. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
3. Give learners an exercise to complete on their own.
4. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://learn.mindset.co.za/resources/mathematics/grade-10/term-1-revision/learn-xtra-live-2013/revision-number-patterns-linear-quadratic-equations

(Revision of number patterns and equations)

Term 1, Topic 5: Topic Overview

TRIGONOMETRY

A. TOPIC OVERVIEW

A

- This topic is the fifth of five topics in Term 1.
- This topic runs for three weeks (13.5 hours).
- It is presented over eight lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 13.5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Trigonometry counts 40% of the final Paper 2 examination.
- Trigonometry is new to learners. The topic includes special angles, solving equations and solving triangles – all important skills for Grade 11 and Grade 12.

Breakdown of topic into 8 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	Introduction	2	5	Solving equations	2
2	Reciprocals	1	6	Solving triangles (2-dimensional problems)	2.5
3	Calculator work	1	7	Cartesian plane and Pythagoras questions	2
4	Special angles	1	8	Revision and Consolidation	2

B SEQUENTIAL TABLE

Senior phase	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
An understanding of: • similar triangles • theorem of Pythagoras	• Definition of the trigonometric ratios ($sin\theta$, $cos\theta$, $tan\theta$) in right-angled triangles • Extend the definitions of the trig ratios to $0° \leq \theta \leq 360°$ • Use the special angles without the use of a calculator • Define the reciprocals of the trig ratios • Solve problems in 2 dimensions	• Derive and use the identities: $tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$ $sin^2\theta + cos^2\theta = 1$ • Derive the reduction formulae • Determine general solutions and specific solutions of trig equations • Establish sine, cosine and area rules • Proof and use of compound angles identities • Proof and use of double angle identities • Solve problems in 2 and 3 dimensions

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Trigonometry.

These include:

- Ignoring the instruction 'without the use of a calculator'
- Algebraic skills (such as using brackets in substitution) let learners down

It is important that you keep these issues in mind when teaching this section.

While teaching Trigonometry, certain basic ideas need to be constantly repeated to learners. These are mentioned within most of the lesson plans.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 1:
  - Investigation/Project
  - Test
- One test, with memorandum, and an investigation, with a rubric are provided in the resource booklet. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of finding sides and angles in right-angled triangles, solving trigonometric equations and calculator work.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

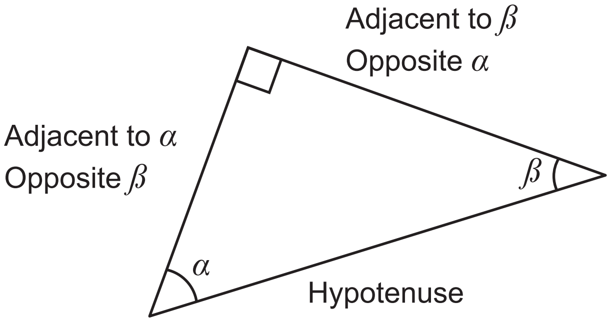
MATHEMATICAL VOCABULARY

E

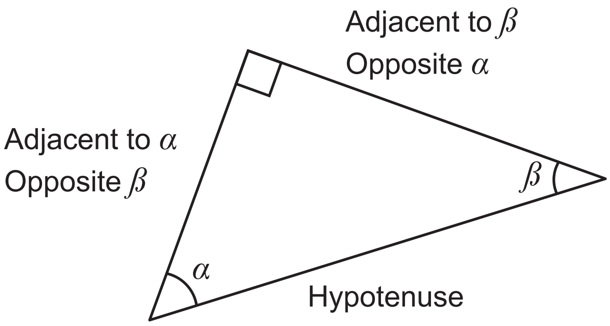
Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
Trigonometry	Trigonometry is concerned with the measurement and calculation of the lengths of sides and the sizes of angles of triangles. Greek letters are often used to stand for angles in triangles.
right-angled triangle	A triangle with one angle of 90^0 (a right angle)
hypotenuse	The longest side in a right-angled triangle. It is always opposite the right angle

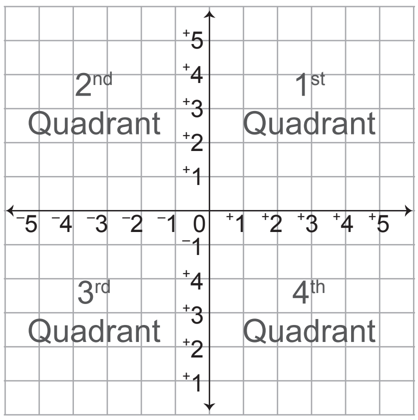
<table>
<tr><td>opposite</td><td>The side opposite any angle you are currently dealing with in a triangle.



</td></tr>
<tr><td>adjacent</td><td>The side next to any angle you are currently dealing with in a triangle.



</td></tr>
<tr><td>quadrants</td><td>The four areas made when a Cartesian plane is drawn with an x-axis and a y-axis.



</td></tr>
<tr><td>reciprocal</td><td>One of a pair of numbers whose product is 1.

$\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$. $\tan\theta$ is the reciprocal of $\frac{1}{\tan\theta}$</td></tr>
</table>

TERM 1, TOPIC 5, LESSON 1

INTRODUCTION

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • name the sides in a right-angled triangle • know the three main ratios.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Have Resource 5 ready for use during lesson.
4. Write the lesson heading on the board before learners arrive.
5. A day or two prior to this lesson, ask learners to draw a right-angled triangle with angles of 60º and 30º. Tell them it is very important that it is an accurate diagram. The size, however, is not important. It can be as small as a cellular phone, as large as a page or anything in between. It should be labelled ABC, where A is the 60º angle, B is the right angle and C is the 30º angle.
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	82	1	86			5.1 5.2	102 105	5.1	112

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Trigonometry is new to learners. This lesson is to introduce them to the topic and help them understand what Trigonometry is all about and why it works.
2. Spend quality time on explanations and answering learners' questions.
3. Tell learners before you start that when a new topic is begun that hasn't been done before, they may be a little confused in the beginning. There are so many explanations and new ideas that it could seem overwhelming. Reassure them that after a few days it will fall into place and start making sense. Remind learners to ask questions when they are unsure.

DIRECT INSTRUCTION

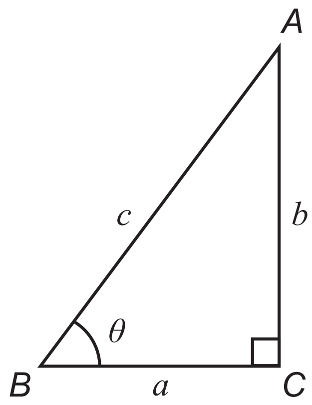
1. Tell learners that there are two topics from previous years which are important in the understanding of Trigonometry. These are, similarity and the theorem of Pythagoras.
2. Say: *Similarity and proportion are important as it is the reason that trigonometry actually works.*
3. Ask: *What does the term similarity mean in mathematics?*
   (Two shapes are similar if they have equal angles and their sides are in proportion).
4. Say: *Although many shapes can be similar, we are particularly interested in triangles. Trigonometry means measurement of triangles. It deals with relations between the sides and angles.*
   When two triangles are similar, their sides are in proportion.

5. Discuss the terms proportion and ratio:
   Proportion: a mathematical concept which states the equality of two ratios. When two sets of numbers increase or decrease in the same ratio, they are said to be directly proportional to each other.
   Ratio: the comparison of the size of two quantities of the same unit.

6. Tell learners that in Grade 10, we only deal with right-angled triangles in Trigonometry.
   In Grade 11, other triangles will be introduced.

7. Spend a minute or two telling learners that although x and y are probably the most commonly used variables, in trigonometry the Greek letters are very common. Write α, β and θ on the board now writing the word next to them (alpha, beta and theta) and tell learners that these will be the most frequently used but there a few others too.

8. Draw a right-angled triangle on the chalkboard. Name one of the acute angles θ.
   Point to angle θ and say: *There always needs to be an angle of interest in the triangle other than the right angle. What I am about to do with you will require our focus to be on this angle.*

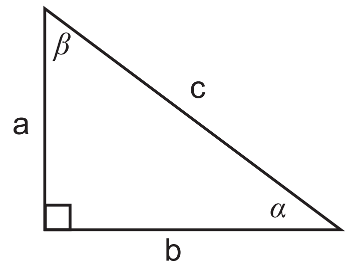


9. Say: *We are going to name the sides of the triangle. Who can tell me what the side is called opposite the right angle?*
   (Hypotenuse).
   Which side is the hypotenuse in this triangle?
   (AB or c).

10. Say: *Now let's name the other two sides. Remember the focus needs to be on the angle θ.*
    Ask: *which side is opposite this angle?*
    (AC or b).
    This side will be called opposite.

11. Ask: *Which side is next to the angle θ?*
    (BC or a).
    This side will be called adjacent as this means next to.

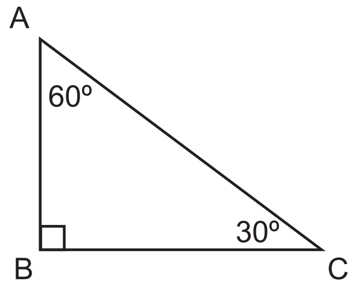
12. Repeat the process with a different triangle and two angles named. Learners must understand that although the hypotenuse is always in the same position (opposite the right angle), the other two sides will change according to the angle named/of interest.



First name the sides according to the angle β: c – hypotenuse; b – opposite; a – adjacent. Erase these labels and name the sides according to α: c – hypotenuse; a – opposite; b – adjacent.

13. Ask learners to take out their calculators. Tell them to find the keys *sin*, *cos* and *tan*. Say: *these have meaning in trigonometry which will be explained soon. Firstly, it is important to note that these are abbreviations.*
sin – sine; cos – cosine; tan - tangent
Tell learners that if they do use the abbreviations when they speak they should rather say sine than sin. (There are no sins in mathematics ☺).

14. Ask learners to take out the triangle they have drawn ready for today. Draw one on the chalkboard according to the instructions given in Classroom Management.



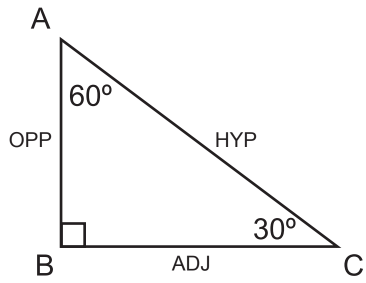
(Remind learners that theirs does not have to look exactly like this one, as long as it is labelled the same way)
Instruct them to measure the length of each side in mm.

15. Once learners have had a few minutes to complete the measuring task, ask them to find the following ratios and round to 3 decimal places. Point out to learners that you know you have repeated two of them but there is a reason for this which they will see later.

$\frac{AB}{AC}$	$\frac{BC}{AC}$	$\frac{AB}{BC}$
$\frac{BC}{AC}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$

16. Once learners have had enough time to find the ratios, tell them to compare their answers with the person next to them.
Ask: *Are your answers the same? If not, are they almost the same?*
The answer should be yes. Tell learners that the only reason they may have different answers is that their measuring may have been inaccurate.

17. Ask learners to name the sides of their triangle based on the 30º angle.



Ask learners to go back to the six ratios given and re-name the first three using the words opposite, adjacent and hypotenuse instead of AB, BC and AC.

$\frac{AB}{AC} = \frac{opp}{hyp}$	$\frac{BC}{AC} = \frac{adj}{hyp}$	$\frac{AB}{BC} = \frac{opp}{adj}$

Before continuing, ask learners to look at their calculators again and check that they are in 'degree' mode. Tell them to always check this, particularly in an assessment.

19. Tell learners that each of these three ratios are directly related to the 30º angle. Ask learners to take out their calculators and press sin 30º. (As learners used decimals, they may need to press the $S \leftrightarrow D$ key to change the fraction to a decimal so it is easier for them to compare).
Ask: *Which answer is closest to this one?*

$\left(\frac{AB}{AC} = \frac{opp}{hyp}\right)$ Say: *sine is always the opposite side divided by the hypotenuse.*

20. Tell learners to cos 30º. Ask: *Which answer is closest to this one?*

$\left(\frac{BC}{AC} = \frac{adj}{hyp}\right)$ Say: *cosine is always the adjacent side divided by the hypotenuse.*

21. Tell learners to press tan 30º. Ask: *Which answer is closest to this one?*

    $\left(\frac{AB}{BC} = \frac{opp}{adj}\right)$ Say: *Tangent is always the opposite side divided by the adjacent side*

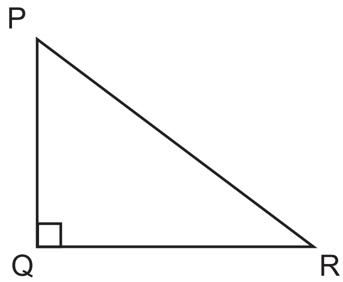
22. Repeat this process with 60º. Learners will need to first rename the sides with the 60º angle being the focus. Repeat the three ratios according to the names of the sides.

23. Once this has been completed, say: *It is important to understand that the trigonometric ratios always work regardless of the size of the triangle. This is because all the triangles are similar – their angles are equal and their sides are in proportion. When you press the key tan 30º, the calculator does not send back a message, 'how big is your triangle?' because the calculator is programmed to know that the angle you are wanting the ratio of is part of a right-angled triangle and that the ratio will be the same regardless of the triangle size.* Repeat this if necessary, asking if anyone has any questions.

24. Make the following statement and ask learners to write it in their books: sine, cosine or tangent of any angle IS A RATIO.

Once learners have started trigonometry they often press keys on their calculator and get the correct answer but don't understand what it is they have actually found. Remind them regularly that the answer they get on their calculator is a ratio. It is one length of one side of a right-angled triangle divided by the length of another side.

25. Draw another right-angled triangle on the chalkboard.



26. Write 50º at R. Ask learners to use their calculators to tell you what sin 50º is. (0,766)

    Ask: *Which sides give the ratio for sine?* $\left(\frac{opp}{hyp}\right)$

    Say: *So, 0,766 is the ratio of this side (point to PQ) divided by this side (point to PR). If we drew this triangle accurately and measured these two sides and divided the length of PQ by the length of PR we should get 0,766 as an answer.*

27. Ask learners to use their calculators to tell you what tan 50º is. (1,192)

    Ask: *Which sides give the ratio for tangent?* $\left(\frac{opp}{adj}\right)$

    Say: *So, 1,192 is the ratio of this side (point to PQ) divided by this side (point to QR). If we drew this triangle accurately and measured these two sides and divided the length of PQ by the length of QR we should get 0,766 as an answer.*

28. Ask learners to use their calculators to find cos 50º is. (0,643)

    Ask: *Which side gives the ratio for cosine?* $\left(\frac{adj}{hyp}\right)$

    Say: *So, 0,643 is the ratio of this side (point to QR) divided by this side (point to PR). If we drew this triangle accurately and measured these two sides and divided the length of QR by the length of PR we should get 0,643 as an answer.*

29. Ask: If $\hat{R}$ = 50º, what is the size of $\hat{P}$?
    (40º).
    Repeat steps 26, 27 and 28 with $\hat{P}$ (40º). Point out the sides again and explain again that if the length of one side was divided by the length of another, that is the ratio given on the calculator.

sin 40º = 0,643	cos 40º = 0,766	tan 40º = 1,192

30. Ask: *Do you notice anything interesting with the answers for the two different angles?*
    (sin 40º = cos 50º and cos 40º = sin 50º).
    Say: *This is an important discovery. Let's look at the triangle to see why this works as it does.*
    Point to $\hat{P}$ (40º). Show, by pointing, that sin 40º = $\frac{QR}{PR}$.
    Point to $\hat{R}$ (50º). Show, by pointing, that cos 50º = $\frac{QR}{PR}$.
    Ask: *Why are the answers the same?*
    (The same sides were being divided).
    Sine and Cosine are called co-ratios – 'co' comes from the word complement which means 'add up to 90º'.

31. Tell learners to test this on their calculators. They can use any two angles that add up to 90º. For example, sin 10º and cos 80º or sin 75º and cos 15º and so on.

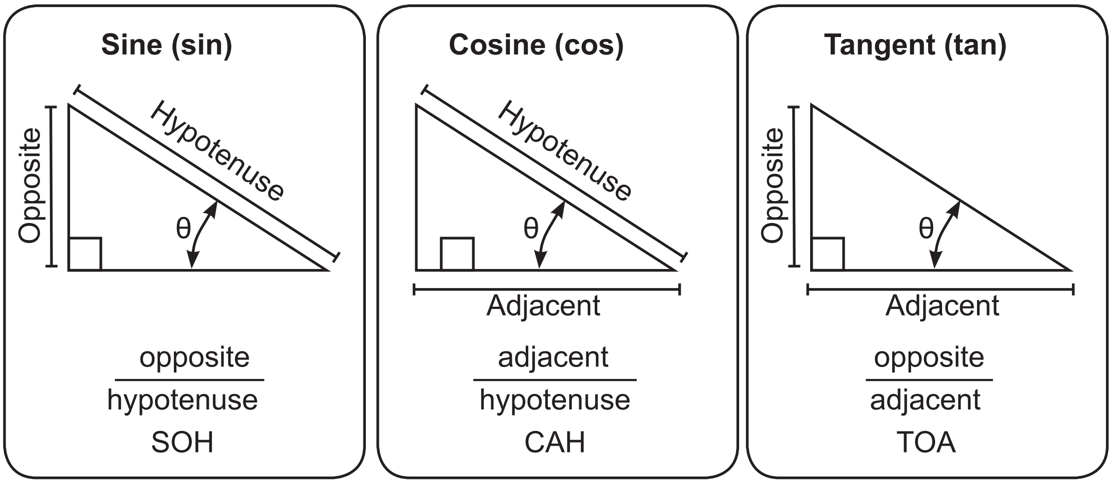
32. Once learners have had chance to test a few pairs of complementary angles, ask:
    Will the sine and cosine of an angle always be less than one? If so, why?
    (Yes, they will, because the sine and cosine of an angle is always a short side divided by a long side because the hypotenuse is always the longest side in a right-angled triangle).

33. Ask: *What about the tangent of an angle?*
    (This could be less than 1 or more than 1. If the opposite side is longer than the adjacent side, it will be more than 1; if the opposite side is shorter than the adjacent side, the ratio will be less than 1).

34. Tell learners that when they do trigonometric functions next term they will be able to visually see that the information in points 32 and 33 is always true.

35. Write the following summary on the chalkboard and ask learners to draw the triangles and write it in their books.
    The mnemonic in the brackets and at the bottom of the diagram is to assist learners in remembering that:
    - **s**in is **o**pposite over **h**ypotenuse (SOH)
    - **c**os is **a**djacent over **h**ypotenuse (CAH)
    - **t**an is **o**pposite over **a**djacent (TOA)

    Point to the letters SOH, CAH and TOA as you explain what the letters represent.
    (This diagram is Resource 5 in the Resource Pack)



36. Before ending, discuss with learners the use of the words sine, cosine and tangent. Remind them that these words don't really make any sense on their own. They only have meaning when used with an angle and then the statement is representing a ratio.

> Tell learners that the only time they should use sine, cosine and tangent on their own (and without an angle) is if they want to say: *I have a sine, cosine and tangent key on my calculator.*

37. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.
38. Give learners an exercise to complete with a partner.
39. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://betterexplained.com/articles/intuitive-trigonometry/

(This video extends the explanation to reciprocals (which will be done later) and some identities (Gr 11) but is well worth a watch. Note however, that the word 'percentage' is used in place of 'ratio' which was used throughout this lesson.

https://www.youtube.com/watch?v=KQhQSd7Wigo

TERM 1, TOPIC 5, LESSON 2

CALCULATOR WORK

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • find ratios on their calculators.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	83	2	88	8.1	84	5.3	110		

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Calculator skills are an important part of mathematics throughout the FET phase.
2. This being said, remind learners that they should not always believe what they see (they could have put something into the calculator incorrectly) – learners should always have an idea what kind of answer to expect and check an answer that does not meet their expectations.

DIRECT INSTRUCTION

1. Before starting the lesson, remind learners what they are finding when the find sine, cosine or tangent of an angle.
   Learners often press keys on their calculator and get the correct answer but don't understand what it is they have actually found. Remind them regularly that the answer they get on their calculator is a ratio. It is one length of one side of a right-angled triangle divided by the length of another side.
2. Tell learners that we are practicing calculator work involving trigonometric ratios. Remind them that their calculator needs to be in degree mode.
3. Say: *In general, many of the calculations that we are about to do, work the same as any algebraic expression that you have put into a calculator before. There is one big difference however. Squaring (or even cubing) a ratio is written differently.*
4. Write the following on the chalkboard:

   $$\cos^2 20^\circ$$

   Say: *This statement actually means: $(\cos 20^\circ)^2$. And although it will be shown like this (point to the original statement) you will need to put it into your calculator like this (point to the 2nd version).*
   Tell learners to find $\cos^2 20^\circ$ now using their calculators. (0,883)
5. Ask: *What have you just found?*
   (The ratio of the length of the adjacent side to 20º divided by the length of the hypotenuse of a right-angled triangle which has then been squared).

6. Do the following examples with learners to ensure they are computing correctly and get the correct answers. Good calculator work is essential to answering trigonometry questions accurately.

7. Tell learners to write them in their books and make notes as they do so.

Use your calculator to determine the value of the following, correct to 2 decimal places.	Teaching notes: Tell learners to close the brackets behind the angles.	Solutions:
$2\sin 20^\circ + 3\cos 10^\circ$	Compute as given but close brackets.	3,64
$\tan(21^\circ + 36^\circ)$	This can be computed as given or learners can find $\tan 57^\circ$.	1,54
$\tan 21^\circ + \tan 36^\circ$	Compute as given but close brackets.	1,11
$\sin^2 25^\circ + \cos^2 25^\circ$	Remind learners what you told them earlier in the lesson and that this is the same as: $(\sin 25^\circ)^2 + (\cos 25^\circ)^2$	1 (Tell learners that this is an important result for later on their trigonometry journey)

8. Ask learners if they have any questions before doing a second set of examples where substitution is necessary first.

9. Tell learners to write them in their books and make notes as they do so.

If a = 35º and b = 72º, use your calculator to find: (round to 3 decimal places)	Teaching notes:	Solutions:
cos $2a$	Once the substitution has taken place, encourage learners to try putting the entire expression into their calculator as well as doing it in a few steps. This will give them more practice on their calculators and more opportunity to find what works and what doesn't work. Brackets play an important role. (Tell learners that the final two answers are an important result for later on their trigonometry journey).	$\cos 2a$ = cos 2(35º) = cos 70º = 0,342
2 cos a		$2\cos a$ = 2 cos 35º = 1,638
sin $2b$		$\sin 2b$ = sin 2(72º) = sin 144º = 0,588
2 $sinb\ cosb$		2 $sinb\ cosb$ = 2 sin 72º cos 72º = 0,588

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=5psyoEDqVhA

TERM 1, TOPIC 5, LESSON 3

RECIPROCALS

Suggested lesson duration: 1 hour

A POLICY AND OUTCOMES

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • list the three reciprocals • do basic calculations using the reciprocals.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5	89	4 (2)	91	8.6	99	5.5	113	5.2	117

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Reciprocals are not a large part of the trigonometry section and are only assessed in Grade 10.

DIRECT INSTRUCTION

1. Start the lesson by asking learners what a reciprocal is and ask for an example. (the reciprocal of a number is 1 divided by that number. For example, the reciprocal of 5 is $\frac{1}{5}$ or the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$)
2. Say: *Today we will be learning about the reciprocals of the trigonometric ratios.*
3. Ask: *What are the three trigonometric ratios that you have already learnt?* (sine, cosine and tangent). Remind learners that the words on their own don't really mean anything and that $\sin\theta$, $\cos\theta$ and $\tan\theta$ would be better answers.
4. Ask: *If we use the definition of a reciprocal, what will the three reciprocals be?*
   $\left(\frac{1}{\sin\theta};\frac{1}{\cos\theta};\frac{1}{\tan\theta}\right)$
5. Say: *Each of these reciprocals have their own names which you will need to learn:*

$\frac{1}{\sin\theta} = \textit{cosecant } \theta$	$\frac{1}{\cos\theta} = \textit{secant } \theta$	$\frac{1}{\tan\theta} = \textit{cotangent } \theta$

6. Tell learners to write the following summary in their books: (This summary is Resource 5 in the Resource Pack)

The six trigonometric functions		
basic	**reciprocal**	
$\sin(\theta) = \frac{o}{h}$	cosecant	$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{h}{o}$
$\cos(\theta) = \frac{a}{h}$	secant	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{h}{a}$
$\tan(\theta) = \frac{o}{a}$	cotangent	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{a}{o}$

7. Tell learners that there are no reciprocal keys on the calculator. When doing calculations with reciprocal functions, learners need to use $\frac{1}{\sin\theta}$; $\frac{1}{\cos\theta}$; $\frac{1}{\tan\theta}$.
8. Do the following examples ,explaining what needs to be put into the calculator. Tell learners to write them in their books and make notes as they do so.

Calculate the following and write your answer to 2 decimal places.	Teaching notes:	Solutions:
sec 20º	Ask: *what is the reciprocal of secant?* (cosine). Say: *change sec 20º to* $\frac{1}{\cos 20^0}$ *and use this for the calculation*	1,06
2 cosec 50º	Ask: *what is the reciprocal of cosecant?* (sine). Say: *change cosec 50º to* $\frac{1}{\sin 50^0}$ *and use this for the calculation.* $2 \times \frac{1}{\sin 50^0} = \frac{2}{\sin 50^0}$	2,61
$\cot^2 67º$	Ask: *what is the reciprocal of cotangent?* (tangent). Say: *change cot 67º to* $\frac{1}{\tan 67^0}$ *and use this for the calculation.* $\left(\frac{1}{\tan 67^0}\right)^2$	0,18

9. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
10. Give learners an exercise to complete with a partner.
11. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=MSoYRaSN_9g

https://www.intmath.com/trigonometric-functions/2-sin-cos-tan-csc-sec-cot.php

TERM 1, TOPIC 5, LESSON 4

SPECIAL ANGLES

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • name the ratios of all the special angles.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**SURVIVAL**		**CLASSROOM MATHS**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	87	3	90	8.5	98	5.9	124	5.3	119

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Even though a calculator can be used to find the special angles, it is important that learners spend some time understanding and therefore being able to learn the ratios of all the special angles.

DIRECT INSTRUCTION

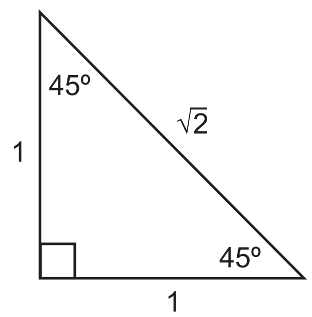
1. Tell learners that they may have already noticed that most angles produce ratios that need to be rounded and therefore cannot be entirely accurate. However, there are a few angles that do produce an exact value when their trigonometric ratio is found. These are the special angles.
2. Say: *Special angles come from special triangles. We are going to explore those now.*
3. Ask learners to draw a right-angled isosceles triangle. It need not be exactly accurate.
   Ask: *What are the sizes of the other two angles?*
   (45º).
   Say: *The key to these calculations is to choose a side length that will be easy to work with. Remember the importance of similar triangles – side lengths will be in proportion.*
4. Let the two shorter and equal sides equal 1. Find the hypotenuse using the theorem of Pythagoras.

$$h^2 = 1^2 + 1^2$$

$$h^2 = 2$$

$$h = \sqrt{2}$$

5. Fill in these measurements on the triangle:



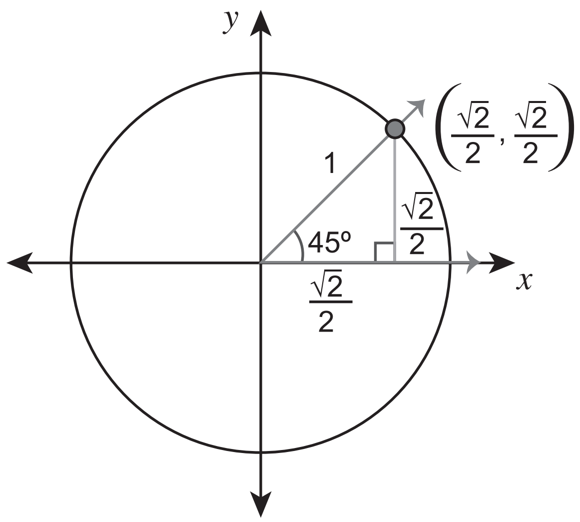
6. Ask learners to write down the ratios of:

$\sin 45^\circ \left(= \frac{1}{\sqrt{2}}\right)$	$\cos 45^\circ \left(= \frac{1}{\sqrt{2}}\right)$	$\tan 45^\circ \left(= \frac{1}{1} = 1\right)$

Although rationalising the denominator is a skill only taught in Grade 11, it would be useful to show it to learners briefly now to see why $\frac{1}{\sqrt{2}}$ is the same as $\frac{\sqrt{2}}{2}$ as given on the calculator.

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$$

Show learners this diagram which shows the triangle represented on the Cartesian plane on what is known as the unit circle.



Point out that the measurements are not the same as the triangle that they have in their books – remind them that the key is that the triangles are similar. The measurements on this triangle will therefore give the same ratios.
Point out to them that the x-coordinate (adjacent) represents the cosine of the angle and the y-coordinate (opposite) represents the sine of the angle. This will be looked at again later in the section.

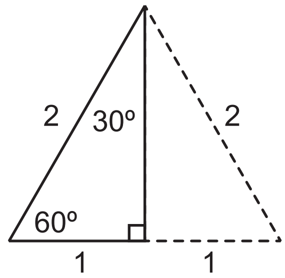
Show learners that $\tan 45^\circ$ will still equal 1 because $\dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

8. Ask learners to draw an equilateral triangle with sides of 2 units each. Tell them it does not have to be accurate.
   Say: *Fill in the side lengths and the angles.*
   Ask: *What are the sizes of the angles?*
   (60º).

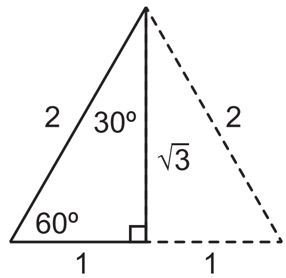
Tell learners to drop a perpendicular from any vertex to the opposite side.

This will create two congruent triangles. Fill in the right angle and the 3rd angle.

Fill in any known sides in one of the new triangles.



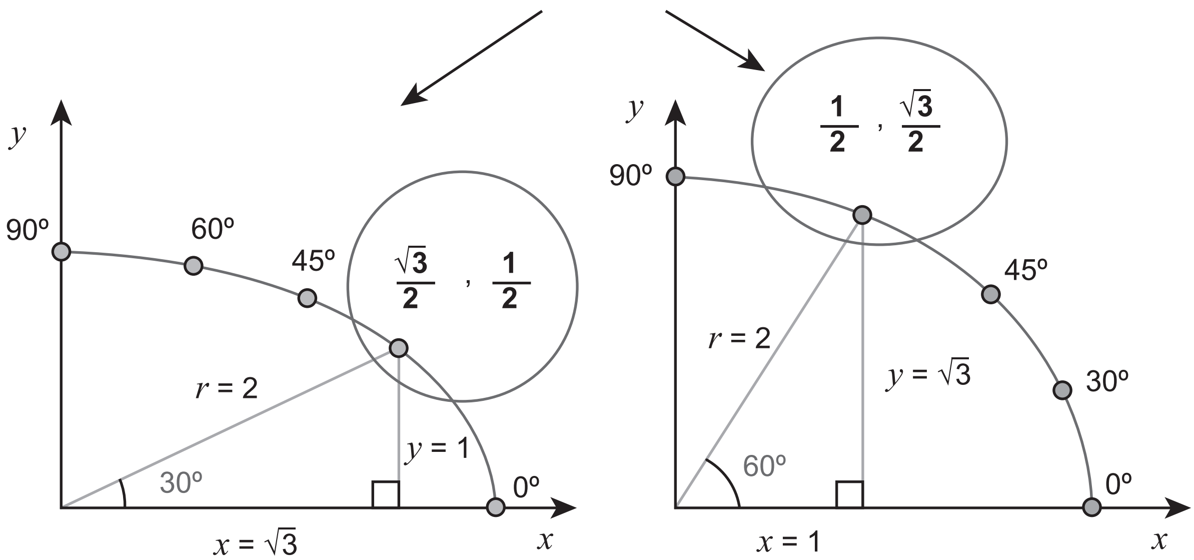
9. Find the missing short side using the theorem of Pythagoras.

$h^2 + 1^2 = 2^2$ $h^2 + 1 = 4$ $h^2 = 3$ $h = \sqrt{3}$	

10. Ask learners to write down the ratios of

sin 60º $\left(= \frac{\sqrt{3}}{2}\right)$	cos 60º $\left(= \frac{1}{2}\right)$	tan 60º $\left(= \frac{\sqrt{3}}{1}\right)$
sin 30º $\left(= \frac{1}{2}\right)$	cos 30º $\left(= \frac{\sqrt{3}}{2}\right)$	tan 30º $\left(= \frac{1}{\sqrt{3}}\right)$

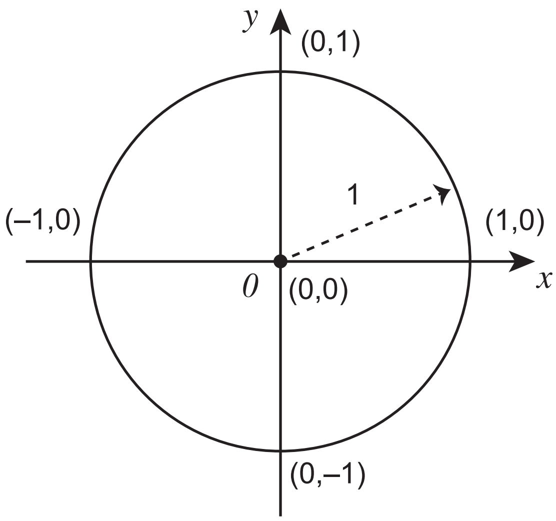
11. Show learners this diagram (It is Resource 6 in the Resource Pack).



Show learners that these triangles are represented in the Cartesian plane on what is known as the unit circle.
Point out that the x-coordinate (adjacent) represents the cosine of the angle and the y-coordinate (opposite) represents the sine of the angle. We look at this again later in the section.

12. Tell learners we want to look at two more special angles — 90º and 0º. These are more difficult to understand as it involves a little bit of imagination. The unit circle will be used.

13. Draw the unit circle on the chalkboard.



14. Remind learners that the 30º triangle would appear first if the radius of 1 was moving anti-clockwise. Draw in a second one representing the 45º and the 60º — the three that have already been discussed. Point out that the moving arm represents the hypotenuse and the height which would also be opposite the angle and is represented by a y-value which would become the y-coordinate at the end of the arm (radius).
The horizontal line (the x-axis) would represent the adjacent side which would become the x-coordinate at the end of the arm (radius).

15. Now ask learners to imagine that the moving arm (which represents the hypotenuse in the right-angled triangle that could be drawn) stops on the y-axis.
Say: *If we stop the arm on the y-axis, we must have formed a right-angled triangle with another right angle in it. This seems impossible, but rather focus on:*
what is the length of the hypotenuse? (1)
and what is the length of the opposite/ y-coordinate? (1)

Therefore, $\sin 90º = \frac{1}{1} = 1$

16. Discuss $\cos 90^\circ$:
    Ask: *Keeping the arm in the same position, at 90º, what is the length of the hypotenuse?* (1) *and what is the length of the adjacent side/x-coordinate?* (0)
    Therefore, $\cos 90^\circ = \frac{0}{1} = 0$
17. Repeat the process to discuss $\cos 0^\circ$ and $\sin 0^\circ$
    Remind learners that the arm (radius) of 1 would start on the x-axis and not leave it in order to be at 0º.
    Ask: *What is the length of the hypotenuse?* (1)
    What is the length of the opposite/ y-coordinate? (0)
    What is the length of the adjacent/x-coordinate? (1)
    Therefore, $\cos 0^\circ = \frac{1}{1} = 1$ and $\sin 0^\circ = \frac{0}{1} = 0$
18. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.
19. Give learners an exercise to complete on their own.
20. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=jI81WXyFrL0
(this video shows a fun way for learners to use their own hand to assist them in remembering the special angles)

TERM 1, TOPIC 5, LESSON 5

SOLVING EQUATIONS

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • find the size of an angle, given the ratio.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write up the three statements from point 1.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	88	8	96	8.2	86	5.4	112	5.6	130

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Solving trigonometric equations is a skill required throughout the FET curriculum.
2. Ensure learners understand each example covered before they try an exercise on their own. If necessary, do further examples from the textbook that your school uses.

DIRECT INSTRUCTION

1. Refer learners to the three statements on the chalkboard. Ask them for the ratios.

sin 30º	tan 45º	cos 90º
$\sin 30^\circ = \frac{1}{2}$	$\tan 45^\circ = 1$	$\cos 90^\circ = 0$

2. Say: *Today we are going to find an unknown angle when given a ratio.*
3. Write the following on the chalkboard:

$\cos\theta = \frac{\sqrt{3}}{2}$	$\sin\alpha = 0$	$\tan\beta = \sqrt{3}$

4. Tell learners that they should recognise that these are all special angles and may already know the answers but that you want to use these to go through the steps on a calculator to find the angle when given the ratio.
5. Ask learners to use their calculators. Say: *The sin, cos and tan keys are used to find the ratios when given the angle which you have been doing so far. To find the angle, we need to use the 2nd function key (shift).*
6. Go through the three examples in full now. Tell learners to write the steps down of which keys to use on the calculator.

	Calculator work	Solution
$\cos\theta = \frac{\sqrt{3}}{2}$	shift/2nd function; cos; $\frac{\sqrt{3}}{2}$	$\theta = 30^\circ$
$\sin\alpha = 0$	shift/2nd function; sin; 0	$\alpha = 0^\circ$
$\tan\beta = \sqrt{3}$	shift/2nd function; tan; $\sqrt{3}$	$\beta = 60^\circ$

7. Ask learners if they have any questions.

8. Before you do the following fully worked examples, point out to learners that calculator work (to find the reference angle) can only take place when the statement is:
   'trig function of an angle is equal to a ratio'. For example, tan $2\alpha = 1{,}43$ is 'ready' for calculator work whereas $2 \sin \theta = 0{,}532$ is not yet 'ready' for calculator work.
   Say: *These will be the first two we look at while working through some fully worked examples together.*

9. Learners should write them in their books and take notes as they do so.

Solve for the unknown angle:	Teaching notes and calculator work
$\tan 2\alpha = 1{,}43$ $\therefore\ 2\alpha = 55{,}03^\circ$ $\alpha = 27{,}52^\circ$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (Yes). Once the reference angle has been found, ask: *What do we still need to do to find α?* (Divide by 2 on both sides).
$2 \sin \theta = 0{,}532$ $\sin \theta = 0{,}266$ $\therefore\ \theta = 15{,}43^\circ$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (No). Ask: *What do we need to do first?* (Divide by 2 on both sides). Ask: *Now is the equation ready to use the calculator and find the reference angle?* (Yes). Say: *In this case, the reference angle is also the solution.*
$\cos(\beta + 10^\circ) = 0{,}765$ $\therefore\ \beta + 10^\circ = 40{,}09^\circ$ $\beta = 30{,}09^\circ$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (Yes). Once the reference angle has been found, ask: *what do we still need to do to find β?* (Subtract 10º from both sides).

$\frac{1}{2}\sin 3\alpha = 0,42$ $\sin 3\alpha = 0,84$ $\therefore\ 3\alpha = 57,14°$ $\alpha = 19,05°$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (No). Ask: *What do we need to do first?* (Multiply by 2 on both sides). Ask: *Now is the equation ready to use the calculator and find the reference angle?* (Yes). Once the reference angle has been found, ask: *What do we still need to do to find α?* (Divide by 3 on both sides).
$2\tan(\theta - 25°) = 2,38$ $\tan(\theta - 25°) = 1,19$ $\therefore\ \theta - 25° = 49,96°$ $\theta = 74,96°$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (No). Ask: *What do we need to do first?* (Divide by 2 on both sides). Ask: *Now is the equation ready to use the calculator and find the reference angle?* (Yes). Once the reference angle has been found, ask: *what do we still need to do to find θ?* (Add 25º to both sides).
$\frac{2\cos(2\beta - 42^0)}{3} = 0,148$ $2\cos(2\beta - 42°) = 0,444$ $\cos(2\beta - 42°) = 0,222$ $\therefore\ 2\beta - 42° = 77,13°$ $2\beta = 119,17°$ $\beta = 59,59°$	Ask: *Is the equation ready to use the calculator and find the reference angle?* (No). Ask: *what do we need to do first?* (Multiply by 3 and divide by 2). Ask: *Now is the equation ready to use the calculator and find the reference angle?* (Yes). Once the reference angle has been found, ask: *what do we still need to do to find β?* (Add 42º to both sides then divide by 2 on both sides).

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
11. Give learners an exercise to complete on their own.
12. Walk around the classroom as learners do the exercise. Support learners where necessary.

TERM 1, TOPIC 5, LESSON 6

SOLVING TRIANGLES (2-DIMENSIONAL PROBLEMS)

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • find an unknown angle or side in a right-angled triangle • answer word problems involving angles of elevation and depression in situations that involve right-angled triangles.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the three triangles from point 7.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
6 7 8	91 92 95	5 6 7 9 10 12	92 93 95 97 98 99	8.4 8.7	93 101	5.6 5.7	116 117	5.4 5.5	124 127

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

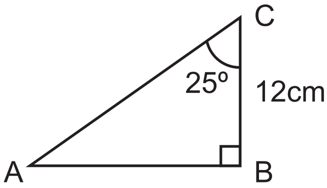
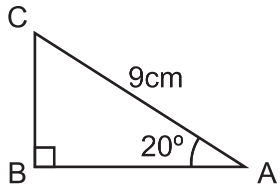
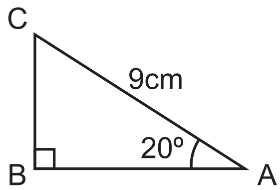
1. Solving triangles is an integral part of Trigonometry.
2. Reading word problems and changing the information into a right-angled triangle diagram is a skill that will be useful in Grade 11 and Grade 12.

DIRECT INSTRUCTION

1. Tell learners that the skill they learned in the previous lesson, solving equations, will be used extensively in this lesson.
2. Say: *Several skills learned in the last week will be combined now to solve triangles.*
3. Explain that solving triangles means to find missing sides or missing angles.
4. Remind learners that the hypotenuse is always across from the right angle and should therefore never be confusing.
5. Tell learners to remember that the focus needs to be on an angle in a triangle (known or unknown) to decide where the opposite and the adjacent is.
6. Share the following information with learners now: To decide which trigonometric ratio to use to help solve a problem, tell learners to always follow these steps/questions:
   - *According to the angle 'named', I have been given the …*
     (opposite, adjacent/hypotenuse)
   - *According to the angle 'named', I am looking for the …*
     (opposite, adjacent/hypotenuse)
   - *Therefore, I will use …* (sine/cosine/tangent)

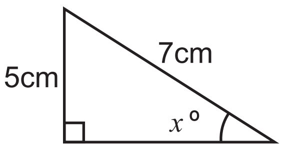
   Learners should write these in their books to refer to later.

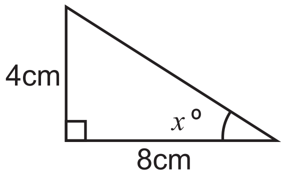
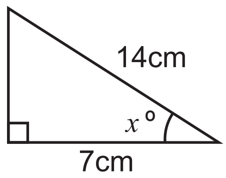
7. Tell learners that we will do fully worked examples now where the side is unknown. They should take them down in their books and make notes as they do so.

	Thought process to choose correct trigonometric ratio	Solution
Find AB 	According to the angle 'named' (25º), I have been given the adjacent According to the angle 'named' (25º), I am looking for the opposite (AB) Therefore, I will use tan	$\tan 25^\circ = \frac{opp}{adj}$ $\tan 25^\circ = \frac{AB}{12}$ 12 tan 25º = AB ∴ AB = 5,6cm
Find AB 	According to the angle 'named' (20º), I have been given the hypotenuse According to the angle 'named' (20º), I am looking for the adjacent (AB) Therefore, I will use cos	$\cos 20^\circ = \frac{adj}{hyp}$ $\cos 20^\circ = \frac{AB}{9}$ 9 cos 20º = AB ∴ AB= 8,46cm
Find BC 	According to the angle 'named' (20º), I have been given the hypotenuse According to the angle 'named' (20º), I am looking for the opposite (BC) Therefore, I will use sin	$\sin 2^\circ = \frac{opp}{hyp}$ sin 20º = $BC/9$ 9 sin 20º = AB ∴ AB = 3,08cm

8. Ask learners if they have any questions.

9. Tell learners that we will do fully worked examples now where the angle is unknown. They should write the examples in their books and make notes as they do so.

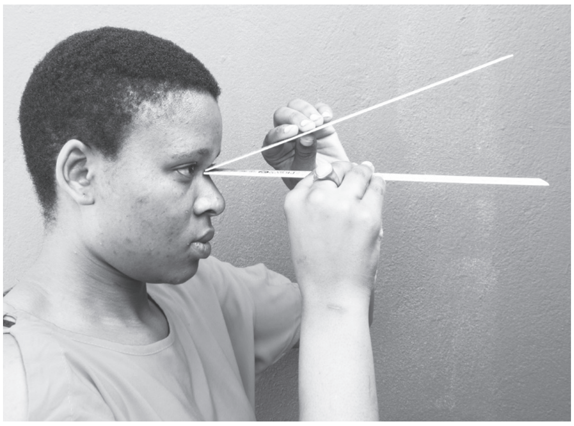
Find the size of the unknown angle (x)	Thought process to choose correct trigonometric ratio	Solution
	According to the angle 'named' (x), I have been given the hypotenuse and the opposite Therefore, I will use sin	$\sin x = \frac{opp}{hyp}$ $\sin x = \frac{5}{7}$ ∴ x = 45,58º (shift ; sin ; $\left(\frac{5}{7}\right)$)

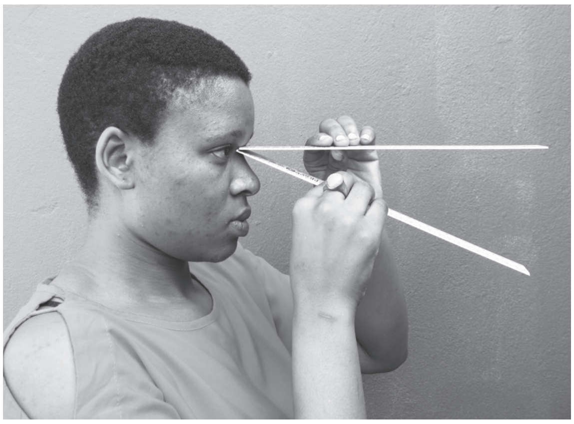
	According to the angle 'named' (x), I have been given the adjacent and the opposite Therefore, I will use tan	$\tan x = \frac{opp}{adj}$ $\tan x = \frac{4}{8}$ $\therefore\ x = 26{,}57^\circ$ (shift ; tan ; $\left(\frac{4}{8}\right)$)
	According to the angle 'named' (x), I have been given the hypotenuse and the adjacent Therefore, I will use cos	$\cos x = \frac{adj}{hyp}$ $\cos x = \frac{7}{14}$ $\therefore\ x = 60^\circ$ (shift ; cos ; $\left(\frac{7}{14}\right)$)

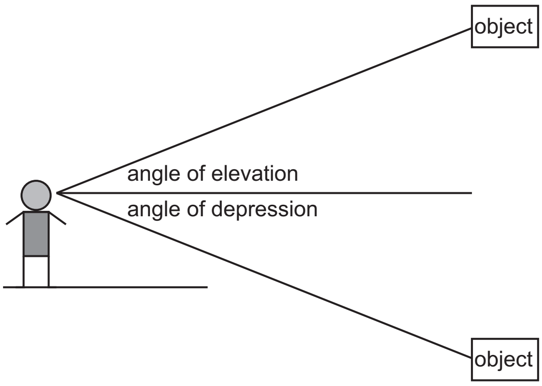
10. Ask learners if they have any questions. Choose a few questions from an exercise in the textbook for learners to confirm they can find an angle and a side of a right-angled triangle using trigonometry.

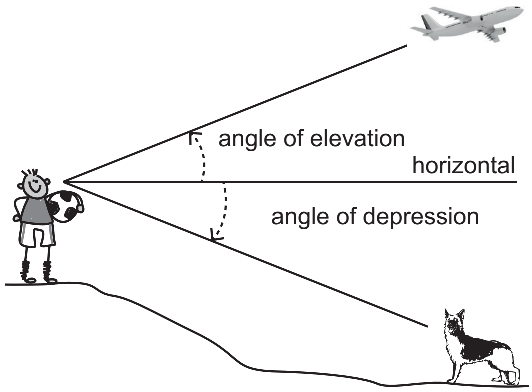
ANGLES OF ELEVATION AND DEPRESSION

11. Say: *These skills can often be tested with word problems. In these cases, the angle of elevation or depression are often involved.*
12. To assist the learners in experiencing this physically, it would be ideal to take them outside.
13. However, first demonstrate the following in the classroom so all learners have had the opportunity to see easily before taking them outside to demonstrate again.
14. You will need two rulers.
15. Hold both rulers (on top of each other) horizontally from the bridge of your nose between the eyes, facing outwards. Ensure they are parallel to the floor.
16. Slowly move the top ruler upwards, while keeping the rulers touching on your nose.
17. The angle formed between the two rulers is an angle of elevation.
18. Explain to learners that the angle is formed when moving from the horizontal in an upwards direction. Tell the learners that you are looking UP at a mark on the ceiling.
19. Repeat, but this time move the bottom ruler downwards for the angle of depression.
20. Explain to learners that the angle is formed when moving from the horizontal in a downwards direction. Tell the learners that you are looking DOWN at a mark on the floor.





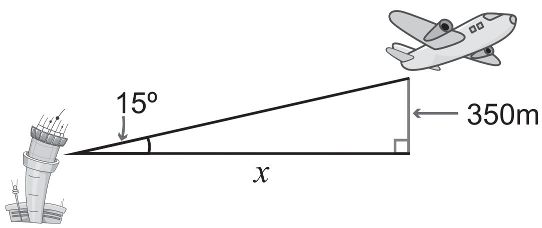


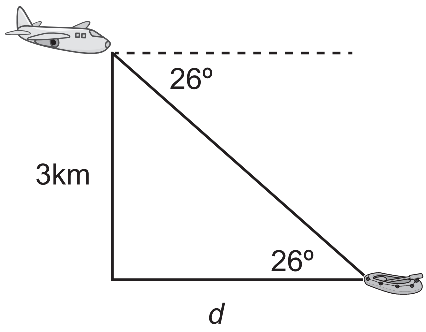


21. Take the learners outside. Ask anyone who has a ruler to bring it with them.
22. Start the lesson outside your own classroom. If your classroom is on the ground floor demonstrate the angle of elevation first. If your classroom is higher up, demonstrate the angle of depression first.
23. Once the first type has been done, take the learners to a suitable place to demonstrate the second one.
24. Demonstrate the angle of depression by asking learners for examples of what object or item they could be looking at on the ground (perhaps a dustbin or a goalpost etc).
25. Demonstrate the angle of elevation by asking learners for examples again of what they could be looking at when looking up (the roof, a window etc).
26. On both occasions allow an opportunity for learners to repeat the motion with the rulers that you have already demonstrated. They can share rulers and have in turns trying it themselves. As they try it themselves, walk around and help them or answer any questions they might have.
27. Once you feel they have all had an opportunity to experience both angle of elevation and angle of depression, take everyone back into the classroom.

28. Ask: *What are the angle of elevation and depression?*
    (Angle of elevation: The angle that a person must look UP to see an object. The angle is formed between the horizontal and the line of sight.
    Angle of depression: The angle that a person must look DOWN to see an object. The angle is formed between the horizontal and the line of sight).

29. Tell learners that these are important concepts and they must ensure they understand them clearly. Ask if anyone has any questions before you do two examples using both types of angles.
    Learners should write the examples in their books and make notes as they do so.

<table>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td>A control tower operator is looking at a plane taking off. If he can see the plane at an angle of elevation of 15º and the plane is 350m above the control tower, what is the horizontal distance from the control tower to the plane?

</td><td>Tell learners to look at the horizontal line and see how the angle is made by moving UP.
Ask: According to the angle named what is given and what is required?
(opposite and adjacent).
Ask: What trig ratio will be used?
(tangent).</td></tr>
<tr><td colspan="2">Solution:
$\tan 15^\circ = \frac{350}{x}$
$x \tan 15^\circ = 350$
$x = \frac{350}{\tan 15^0}$
$\therefore\ x = 1306,22\text{m}$</td></tr>
</table>

Example	Teaching notes
An airplane pilot sights a life raft at a 26º angle of depression. The airplane's altitude is 3km. What is the airplane's surface distance d from the raft? 	Tell learners to look at the horizontal line and see how the angle is made by moving DOWN. Remind learners that the horizontal line of sight is parallel to the ground and it should therefore be clear that the angle at the raft (inside the triangle) is also 26º due to alternate angles being equal. Ask: *According to the angle named what is given and what is required?* (opposite and adjacent). Ask: *What trig ratio will be used?* (tangent).
Solution: $\tan 26º = \frac{3}{d}$ $d \tan 26º = 3$ $d = \frac{3}{\tan 26^0}$ $\therefore\ x = 6{,}15\text{m}$	

30. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

31. Give learners an exercise to complete on their own.

32. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=SbV_W9r5dDI

https://www.youtube.com/watch?v=fjUN809FPRs

https://www.youtube.com/watch?v=w4D4KOxFsKA

TERM 1, TOPIC 5, LESSON 7

CARTESIAN PLANE AND PYTHAGORAS QUESTIONS

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners should be able to: • find the value of trigonometric ratios given information and without the use of a calculator.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises. Find a stick or cut out a piece of card to use as the 'arm' to show learners different angles in the Cartesian plane.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
9 10	99 100	4(1)	91	8.3	90	5.8	120	5.7	136

CONCEPTUAL DEVELOPMENT

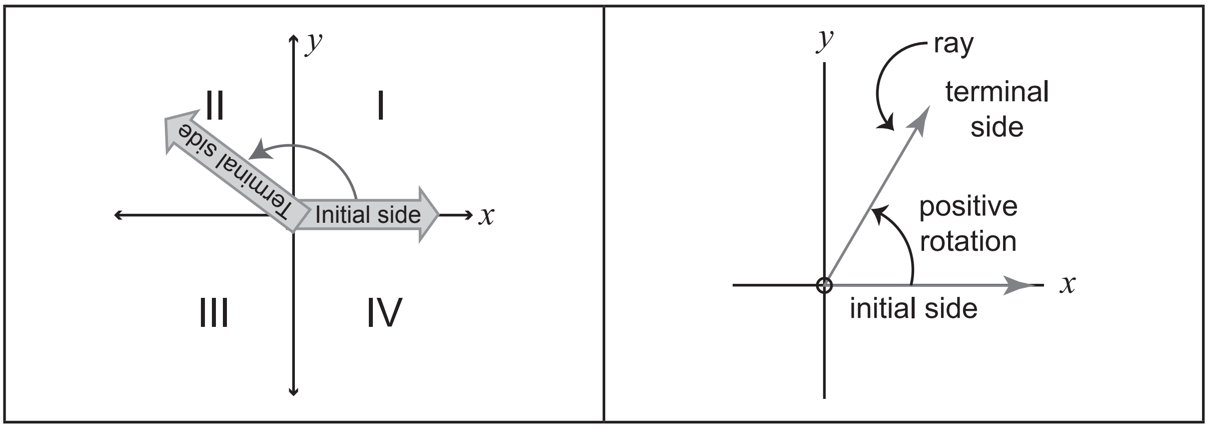
INTRODUCTION

1. Understanding that there can be angles in any quadrant is a key point to a deeper understanding of trigonometry both now and in future grades.

DIRECT INSTRUCTION

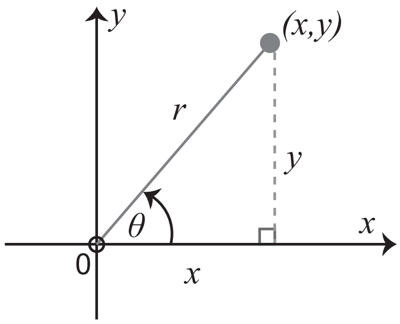
1. Start the lesson by referring to the Cartesian plane drawn on the chalkboard. Make sure you have the item you planned before the lesson to use as the ‘arm’ on the Cartesian plane (the word ‘stick’ will be used from now on).

2. Using the stick, place it at the origin and lay it along the x-axis. Tell learners that this is the starting point for all angles: 0º. Point out that all positive angles are made by moving in a clockwise direction. Move the stick slowly and stop somewhere in the 1st quadrant.



Note: learners should write notes, as well as draw the diagrams, discussed in points 3 to 15.

3. Show learners that this angle is an acute angle. Remove the stick and draw in a perpendicular to form a right-angled triangle (as shown on next page). Discuss with learners why the distance along the x-axis must be represented by an x-value and the distance of the perpendicular dropped must be represented by a y-value. If need be, use the stick to show again that as you rotate you are essentially forming the unit circle and the stick represents the radius and this is the reason it is labelled r.



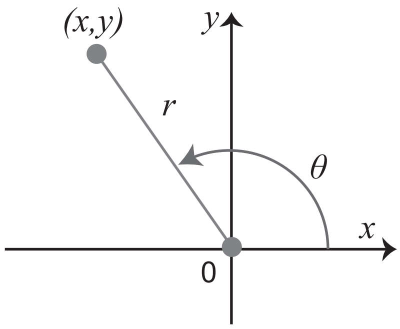
4. Discussw the coordinate and how it would give us the value of both the x and the y distances on the triangle.

5. Ask: *Which sides represent the opposite and adjacent?*
   (The opposite is y and the adjacent is x).
   Ask learners to use x,y and r and write down the ratios for sin θ,cos θ and tan θ.

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$

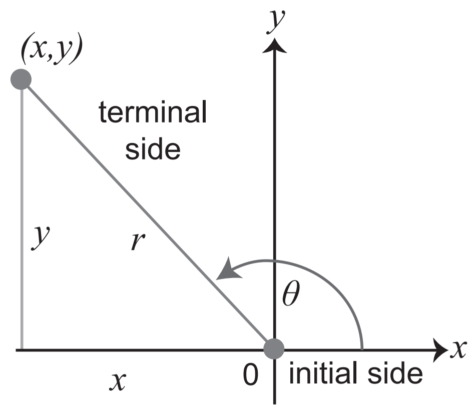
6. Ask: *If we knew two of the three lengths, what could we use to find the 3rd side?*
   (Theorem of Pythagoras).

7. Use the stick again to start at 0° and rotate until the 2nd quadrant (as in the diagram below).



   Ask: *what type of angle has been formed now?*
   (Obtuse).

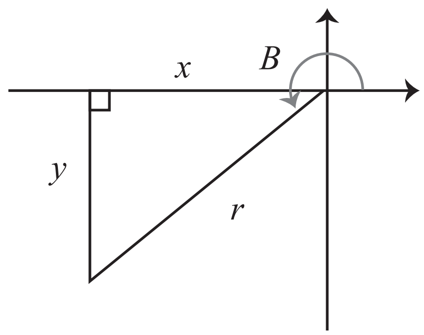
8. Drop a perpendicular to form a right-angled triangle.



Fill in the x, y and r.

9. Say: *Although x, y and r all represent lengths on the diagram, this is not the case for the coordinate.*
   Ask: *Which variable would be negative in this case?*
   (x).
   Point out that is we were to use any ratios that involved x (the adjacent side), that these ratios would then be negative.
   Ask: *Which ratios would be negative for an obtuse angle and therefore in quadrant 2?*
   (cosine and tangent).

10. Use the stick again to start at 0º and rotate until the 3rd quadrant (as in the diagram below).



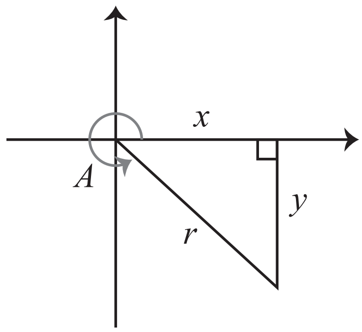
Ask: *What type of angle has been formed now?*
(Reflex).

11. Draw in a perpendicular to form a right-angled triangle.
    Fill in the x, y and r.

12. Say: *Although x, y and r all represent lengths on the diagram, this is not the case for the coordinate.*
    Ask: *Which variable(s) would be negative in this case?*
    (x and y).

Point out that if we were to use any ratios that involved x (the adjacent side) or y (the opposite side) that this could affect the sign of the ratios.
Ask: *Which ratios would be negative for a reflex angle in quadrant 3?*
(cosine and sine).
Tangent would not be negative because $\frac{-}{-} = +$

13. Use the stick again to start at 0º and rotate until the 4th quadrant (as in the diagram below).



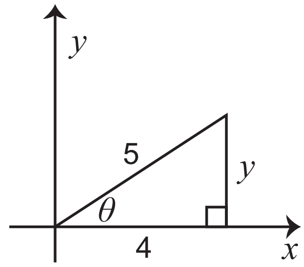
Ask: *What type of angle has been formed now?*
(Reflex).

14. Draw in a perpendicular to form a right-angled triangle.
Fill in the x, y and r.

15. Say: *although x, y and r all represent lengths on the diagram, this is not the case for the coordinate.*
Ask: *Which variable(s) would be negative in this case?*
(y).
Point out that if we were to use any ratios that involved y (the opposite side) that this could affect the sign of the ratios.
Ask: *Which ratios would be negative for a reflex angle in quadrant 4?*
(sine and tangent).

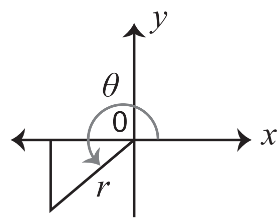
16. Time permitting, allow learners a few minutes to test these statements on their calculator. They should think of an obtuse angle (for example, 135º) and find the sine, cosine and tangent of the angle and check that their answers are positive or negative according to what they have just learners. This can be repeated with a reflex angle between 180º and 270º (quadrant 3) and again with reflex angles between 270º and 360º (quadrant 4).

17. Do fully worked examples of the type of question they could be asked to assess this concept. They should write them in their books and make notes as they do so.

Question	Teaching notes	Solution
If $\cos\theta = \frac{4}{5}$ and $0° \le \theta \le 90°$, determine the value of: a) $\sin\theta$ b) $\tan\theta$ c) $(\tan\theta + \cot\theta)$ d) $\cos^2\theta$	Step 1: Ensure the information given about the angle is in its simplest form. It is. Step 2: Using the information from step 1 AND the second piece of information about the angle decide which quadrant the angle lies in. 1st piece of information: cos of the angle is positive, ∴ the angle must be in quadrant 1 or 4 2nd piece of information: the angle lies in quadrant 1 ∴ the angle must be in quadrant 1 as it is the only one in common. Step 3: Draw the triangle in the correct quadrant. Tell learners to imagine a bowtie crossing at the origin to ensure they draw the triangle correctly. Step 4: Fill in the 2 known lengths from step 1 (in this case the adjacent and the hypotenuse) Step 5: Use the theorem of Pythagoras to find the missing length Step 6: Make a summary of the values for x, y & r.	Step 2: Quadrant 1 Steps 3 and 4:  Step 5: $y^2 + 4^2 = 5^2$ $y^2 + 16 = 25$ $y^2 = 9$ $y = 3$ Step 6: $x = 4$ $y = 3$ $r = 5$ Step 7: a) $\sin\theta = \frac{3}{5}$ b) $\tan\theta = \frac{5}{4}$ c) $\tan\theta + \cot\theta$ $= \tan\theta + \frac{1}{\tan\theta}$ $= \frac{5}{4} + \frac{1}{\frac{5}{4}}$ $= \frac{5}{4} + \frac{4}{5}$ $= \frac{25}{20} + \frac{16}{20}$ $= \frac{41}{20}$ d) $\cos^2\theta$ $= \left(\frac{4}{5}\right)^2$ $= \frac{16}{25}$

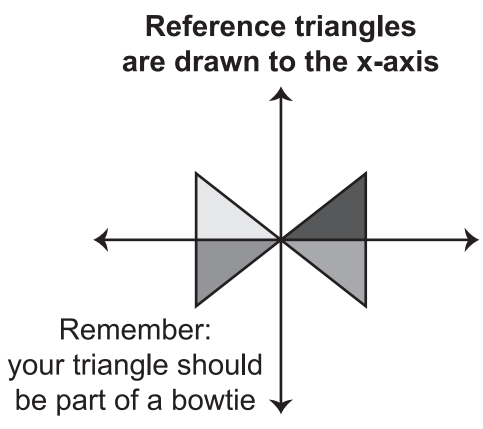
	NB: Now learners need to take careful note of whether x or y could be positive (r is ALWAYS positive). An error here will change their final answer! For this example, everything is positive due to the angle being in quadrant 1. Discuss the potential problem with learners anyway. Step 7: Use the summary from step 6 to substitute and answer the actual question. Remind learners that the 'square' is in an odd place in trigonometry and that $cos^2\,\theta = (cos\,\theta)^2$ when substituting actual values.	

18. Ask learners if they have any questions before doing a second example.

Question	Teaching notes	Solution
If $5 \sin\theta + 4 = 0$ and $0° \leq \theta \leq 270°$, calculate the value of $\sin^2\theta + \cos^2\theta$.	Step 1: Ensure the information given about the angle is in its simplest form. Step 2: Using the information from Step 1 AND the second piece of information about the angle decide which quadrant the angle lies in. First piece of information: sin of the angle is negative, ∴ the angle must be in quadrant 3 or 4 Second piece of information: the angle lies in quadrants 1, 2 or 3 ∴ the angle must be in quadrant 3 as it is the only one in common.	Step 1: $5\,sin\,\theta + 4 = 0$ $5\,sin\,\theta = -4$ $sin\,\theta = \frac{-4}{5}$ Step 2: Quadrant 3 Steps 3 and 4:  Step 5: $x^2 + 4^2 = 5^2$ $x^2 + 16 = 25$ $x^2 = 25 - 16$ $x^2 = 9$ $x = 3$

	Step 3: Draw the triangle in the correct quadrant. Tell learners to imagine a bowtie crossing at the origin to ensure they draw the triangle correctly. Step 4: Fill in the two known lengths from Step 1 (in this case the opposite and the hypotenuse) Tell learners they can ignore the negative at this stage as we are dealing with lengths. Step 5: Use the theorem of Pythagoras to find the missing length Step 6: Make a summary of the values for x, y & r. NB: Now learners need to take careful note of whether x or y could be positive (r is ALWAYS positive). An error here will change their final answer! Step 7: Use the summary from Step 6 to substitute and answer the actual question.	Step 6: $x = -3$ $y = -4$ $r = 5$ Step 7: $sin^2\theta + cos^2\theta$ $= \left(\frac{-4}{5}\right)^2 + \left(\frac{-3}{5}\right)^2$ $= \frac{16}{25} + \frac{9}{25}$ $= \frac{25}{25} = 1$

19. To assist learners in drawing the triangles in the quadrants, make a sketch like this on the chalkboard:



20. Ask directed questions so that you can ascertain learners' level of understanding.
Ask learners if they have any questions.

21. Give learners an exercise to complete with a partner.

22. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=50BLDkLP2Is

https://www.youtube.com/watch?v=oaekbKvdO_Y

TERM 1, TOPIC 5, LESSON 8

REVISION AND CONSOLIDATION

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	23
Lesson Objectives By the end of the lesson, learners will have revised: ● all the concepts covered in this section.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson have the first few questions ready.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev S Ch	112 116	Rev	107	w/sh	107	5.11 5.12	128 130	5.8	138

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

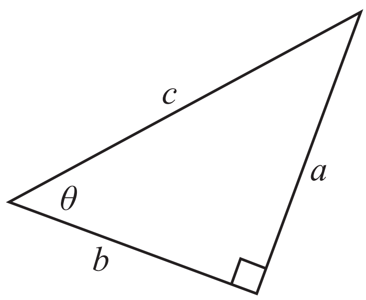
This lesson is made up of fully worked examples from a past examination covering most of the concepts in this topic. As you work through these with the learners, it is important to frequently talk about as many concepts as possible.

For example, use the words ratio, reciprocal, special angles, opposite, adjacent and hypotenuse.

Say: *I am going to do an entire Trigonometry question from the 2016 final examination with you. You should write it down as I do them, taking notes at the same time.*

Example:

1. A right-angled triangle has sides a, b and c and the angle θ, as shown below:



   a) Write the following in terms of a, b and c:
      i) $\cos\theta$
      ii) $\tan\theta$
      iii) $\sin(90° - \theta)$
   b) If it is given that $a = 5$ and $\theta = 50°$, calculate the numerical value of b.
2. Given that $\hat{A} = 38{,}2°$ and $\hat{B} = 146{,}4°$. Calculate the value of $2\,cosec A + \cos 3B$.
3. Simplify fully, WITHOUT the use of a calculator:

$$\frac{\sin 45^0 . \tan^2 60^0}{\cos 45^0}$$

4. Given that $5\cos\beta - 3 = 0$ and $0° < \beta < 90°$.
   If $\alpha + \beta = 90°$ and $0° < \alpha < 90°$, calculate the value of $\cot\alpha$.

Teaching notes:

1a) Learners should find (i) and (ii) simple if they know their ratios.
For (iii), discuss angles of a triangle with learners.
Ask: *If this angle (point to it) is θ, what is the size of the other angle (point to it).*
If learners struggle to see that the other angle is ($90° - \theta$), use a few actual values to show them that the 3rd angle will be 90º — the given value.

b) Once the substitution has been made, this should be a simple calculation (providing learners know their ratios).
Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*

2. Tell learners that this is a substitution and calculator work question but they also need to know their reciprocals. Ask: *What is the reciprocal of cosecant?*
(sine).

3. Remind learners that if they are instructed to not use a calculator they should not be doing so. Ask: *What knowledge is required for this question?*
(Special angles).

4. Tell learners they first need to simplify the statement given so that they have a statement for cos β. Once this has been done they can use their calculator to find the value of β. This will then be enough information to use the statement given to find α. Finally, this needs to be used to substitute and find cot α.
Ask: *What is the reciprocal of cotangent?*
(tangent).

Solutions:

1. a)

(i) $\frac{b}{c}$	(ii) $\frac{a}{b}$	(iii) $\frac{b}{c}$

b)
$$\tan\theta = \frac{a}{b}$$
$$\tan 50° = \frac{5}{b}$$
$$b\tan 50° = 5$$
$$b = \frac{5}{\tan 50^0}$$
$$b = 4{,}2$$

2. $2\,cosec\,A + cos\,3B$
$= 2\operatorname{cosec} 38{,}2° + \cos 3(146{,}4°)$
$= 2\left(\frac{1}{\sin 38{,}2^0}\right) + \cos 3(146{,}4°)$
$= 3{,}42$

3. $\dfrac{\sin 45^0 . \tan^2 60^0}{\cos 45^0}$

$= \dfrac{\frac{1}{\sqrt{2}} . \left(\frac{\sqrt{3}}{1}\right)^2}{\frac{1}{\sqrt{2}}}$

$= \left(\dfrac{\sqrt{3}}{1}\right)^2$

$= 3$

4. $\cos\beta = \dfrac{3}{5}$

$\therefore \ \beta = 53{,}13^\circ$

$\alpha + \beta = 90^\circ$

$\alpha = 36{,}87^\circ$

$\cot\alpha$

$= \dfrac{1}{\tan\alpha}$

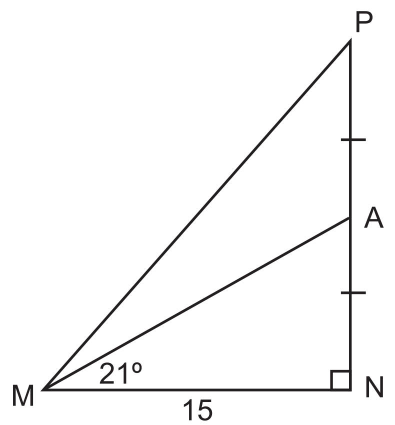
$= \dfrac{1}{\tan 36{,}87^0}$

$= 1{,}33$

Example:

1. In the sketch below, ΔMNP is drawn having a right angle at N and MN = 15 units.
   A is the midpoint of PN and $A\hat{M}N = 21^\circ$

   Calculate:
   a) AN
   b) $P\hat{M}N$
   c) MP



2. Calculate θ if $2\sin(\theta + 15^\circ) = 1{,}462$ and $0^\circ \leq \theta \leq 90^\circ$

Teaching notes:

1 a) Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*

b) Ask: *Which triangle will we need to work in now? (∆PMN). Tell learners to fill in any information that is not yet on the diagram (AN and AP)*

Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*

c) Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*

(Note: there are a few options for this question, including the theorem of Pythagoras. Encourage learners to use trigonometry as it is new and still needs practice. Point out, however, that in an assessment they could use any method that is mathematically correct).

2. Ask: *Is the equation ready to use the calculator and find the reference angle?* (No).

Ask: *What do we need to do first?*

(Divide by 2 on both sides).

Ask: *Now is the equation ready to use the calculator and find the reference angle?* (No).

Once the reference angle has been found, ask: what do we still need to do to find θ? (Subtract 15º from both sides).

Solutions:

1. a) In $\triangle AMN$:

$$\tan 21^\circ = \frac{AN}{15}$$

$$15\tan 21^\circ = AN$$

$$5{,}76 = AN$$

$$AN = 5{,}76 \text{ units}$$

b) In ∆PMN:

$$\tan \hat{M} = \frac{PN}{MN}$$

$$\tan \hat{M} = \frac{2(5{,}76}{15}$$

$$\therefore \text{ M} = 37{,}52^\circ$$

$$\therefore P\hat{M}N = 37{,}52^\circ$$

c) $\cos 37{,}52^\circ = 15/MP$

$$MP\cos 37{,}52^\circ = 15$$

$$MP = \frac{15}{\cos 37{,}52^0}$$

$$MP = 18{,}91$$

2. $2\sin(\theta + 15^\circ) = 1{,}462$

$$\frac{2\sin(\theta + 15^0)}{2} = \frac{1{,}462}{2}$$

$$\sin(\theta + 15^\circ) = 0{,}731$$

$$\therefore\ \theta + 15^\circ = 46{,}97^\circ$$

$$\theta = 31{,}97^\circ$$

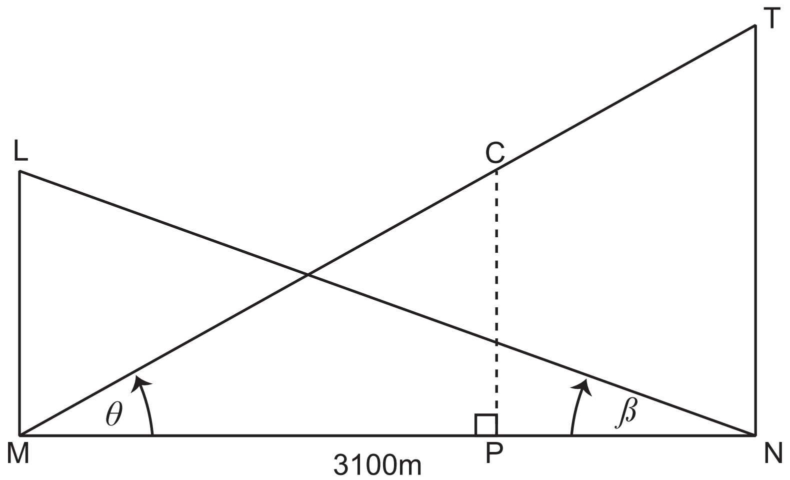
Example:

The diagram below represents a cross-section of the peaks of Table Mountain, T, and Lions Head, L, above sea level. Points M and N are directly below peaks L and T respectively, such that MPN lies on the same horizontal plain at sea level and P is directly below C.

MN = 3 100m.

The angle of elevation of L from N is β and the angle of elvation of T from M is θ.

It is given that tan θ = 0,35 and tan β = 0,21.



1. Calculate the ratio of $LM{:}TN$.
2. A cable car, C, travelling from the top of Table Mountain, T, follows a path along TCM.
   a) Calculate the angle formed ($M\hat{T}N$) between the cable and the vertical height TN.
   If the cable car, C, travels along the cable, such that TC =400m, calculate the height of the cable car above sea level at that instant.

Teaching notes:

1. To answer this question requires finding the lengths of LM and TN.
   First ask learners to focus on LM. Ask: *Which triangle will we need to work in?* (ΔLMN)
   Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*
   Point out to learners that they have already been given the ratio for tan β.
   Now ask learners to focus on TN. Ask: *Which triangle will we need to work in?* (ΔTNM)
   Ask: *According to the angle named, I have been given the…? And am looking for…? Therefore, I need to use…*
   Point out to learners that they have already been given the ration for tan θ.
   Once the two measurements have been found, the ratio can be found.

2a) Ask: *According to the angle required, I have the….and the …? Therefore, I will use…?*

b) Show learners that TC is not part of a triangle, but TM is which we can find. Once that has been found, 400 can be subtracted which will leave us with the measurement of MC. This is in a right-angled triangle, so we can find PC.

Solutions:

1. $\frac{LM}{MN}$ = tan β = 0,21

$\frac{LM}{3100}$ = 0,21

$\therefore LM$ = 651m

$\frac{TN}{MN}$ = tan θ = 0,35

$\frac{TN}{3100}$ = 0,35

$\therefore TN$ = 1085m

$\therefore \frac{LM}{TN} = \frac{651}{1085} = \frac{3}{5}$

$\therefore LM:TN$ = 3:5

2. a) tan $\hat{T} = \frac{3100}{1085}$

$\therefore \hat{T}$ = 70,71º

$\therefore M\hat{T}N$ = 70,71º

b) θ = 19,29º (<'s of Δ)

cos 19,29º = $\frac{3100}{TM}$

TM = 3284,39

TC = 400

$\therefore CM$ = 3284,39-400 = 2884,39

$\therefore$ sin 19,29º = $\frac{CP}{2884,39}$

CP = 952,86m

1. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
2. Give learners an exercise to complete on their own.
3. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://learn.mindset.co.za/resources/mathematics/grade-10/term-1-revision/learn-xtra-live-2013/revision-trigonometry